Geometric Pattern Design in Islamic Architecture Using Computer Graphics

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Abstract

Throughout the history of Islam architecture, eminent craftsmen have adorned buildings with Islamic geometric patterns IGP. The IGP designs are considered the most recognizable expression of Islamic art and architecture and occur in rich profusion throughout Islamic cultures. The IGP attributes that are used to construct the designs have a special place and important hidden meanings in each civilization, culture, and religion. Today we have unparalleled ability to understand the traditional IGP of the past, and to innovate new designs.

Mathematical, algorithmic, and technological tools if taken together provide us with new opportunities to analyse and explore IGP designs. The mathematical tool enables us to illustrate the original designs. The algorithmic tools enable us to perform calculations with the help of computers. Finally, technological tools enable us to turn the designs into a real artifact. This thesis is a research study in computer-generated IGP designs.

The research is focused on two areas in particular; the study of several significant design methodologies that advocate IGP designs and arrive at a different way of setting the algorithm based on IGP motif to develop new designs with ease. The second area is the classification of IGP with normalization grid methodology rather than the classification based on 7-frieze and 17- research wallpaper group theories.

Also, in this thesis, we will shed light on the art sacred meanings in every civilization, culture, and religion with the emphasis on system grid attributes, and illustrates how modern mathematics, algorithms, and technology can be applied to the study of IGP designs.

Dedication

To my parents to whom I owe everything I have accomplished in my life. To my brothers, sisters, wife, and children for all their love and support, without it I would not be able to complete this thesis.

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List of Publications and Communications

Publications

1. Ahmad Aljamali and Ebad Banissi: Normalised Grid/Motif Based Patterns Islamic Geometric Patterns, IEEE Conference, Information Visualization (IV 1997), Pages: 252-257. ISBN: 0-8186-8076-8, http://www.computer.org/csdl/proceedings/iv/1997/8076/00/80760252-abs.html

2. Ahmad Aljamali and Ebad Banissi: Normalisation and Exploration Design Method of Islamic Geometric Patterns, International Conf. on Geometric Modelling & Graphics, London, U.K. Published by the IEEE Computer Society, Pages: 42-48 (2003). ISBN: 0769519857 http://www.computer.org/csdl/proceedings/gmag/2003/1985/00/19850042-abs.html

3. Ahmad Aljamali and Ebad Banissi: Grid Method Classification of Islamic Geometric Patterns, The 11th WSCG Conference, University of West Bohemia, Plzen, Czech Republic, UNION Agency – Science Press, pages: 17-24 (2003) ISBN 80-903100-1-X. http://wscg.zcu.cz/wscg2003/Papers_2003/K53.pdf

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5. Ahmad M. Aljamali and M Roberts Masillamani: Classification and Design of Islamic Geometric Patterns using computer graphics, Jupiter Publications Consortium, International Journal of Research in Mechanical, Mechatronics and Automobile Engineering, Vol. 3 issue 2, pp. 13-24, 15 Nov 2017. ISSN Print: 2454-1435, ISSN Online: 2454-1443. http://ijrmmae.in/Volume3-Issue-3/paper2.pdf

6. Ahmad Aljamali & M. Fakir: An Expert System for The Design and Classification of Islamic Geometric Patterns Using Computer Graphics. design patterns, Journal of Theoretical and Applied Information Technology 15 January 2019. Vol.96. No 1 – ongoing JATIT & LLS ISSN: 1992-8645 www.jatit.org E-ISSN: 1817-3195. http://www.jatit.org/volumes/Vol97No1/26Vol97No1.pdf

Communications

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2. Ahmad Aljamali & M. Fakir: An Expert System Islamic Design Patterns, Computer Vision, Pattern Recognition (CBI'17), March 29 - 31, 2017, Beni Mellal, Morocco.

3. Ahmad Aljamali: Classification of Geometric Islamic Patterns by Computer Graphics, CBI’18, Beni Mellal, 2018.

Contents

Chapter 1: Introduction

The history of art decorations reflects human venture. From the moment humans began to build objects, they decorated them with patterns. One of these decoration styles is Islamic geometric patterns IGP, which occur vastly throughout Islamic cultures. More than a thousand years ago, Islamic artisans began to adorn architectural surfaces with IGP and functions as the major decorative element on a vast array of objects of all types. The most striking characteristics of IGP are the IGPs. This practice developed into a rich Islamic art that followed the spread of Islamic culture into Asia, Africa, and Europe. The significant intellectual contributions of Islamic mathematicians, astronomers, and scientists were essential for the creation of IGP designs. Islamic artists reserved the traditional design methodologies, and later elaborated on their methods to create new designs that show the importance of unity and order. At the beginning of the Middle Ages, the scientific interest in the field of geometric patterns was clear.

The works of Euclid and Pythagoras were among the earliest translated into Arabic. Theorists presented their admiration for IGP in the 7-frieze and 17- wallpaper group theories. This led us to analyse the works of eminent experts like Xah Lee [1], Grunbaum Branko [2], Andrew Glassner [3], and Doris Schattschneider [4] to get a holistic view of the entire spectrum of work on this subject.

In the last century, technology enabled us to develop mathematical tools that help us to analyse IGP designs. The concept of designs crossed over to the computer science field through the efforts of the programming community. Interactive tools give the artist instant feedback on their work. The advancements in computer technology and programming provided scientists, mathematicians, and artists powerful research tools. The computer has become a tool for artistic exploration, with little fear of committing unfixable errors or wasting resources.

Questions related to the history of designs have established researchers' curiosity to discover information about how they originally been designed and how the pioneer designers explained their hidden sacred meanings. The IGP designs are popularly associated with Islamic art, largely due to their iconic quality. The urge to decorate the architectural surfaces with IGP designs bound up with the Muslim artist's beliefs and spiritual conditions. Driven by religious passion, the Muslim intellectuals recognized their importance as the unifying intermediary between the material and the spiritual world. The study of stars and astronomy by Muslim scientists in early Islamic civilization led to a passionate interest in IGP designs.

The objective of this thesis is to concentrate on two areas of research: classification and design of IGP. The goal is to achieve devising algorithms for various IGP designs and turning those models into software based on their normalization grid methodology. From the survey of related works and literature, this research addresses two main problems.

Problem 1: The fundamental principles of IGP classification by mathematicians used the 7-frieze and the 17-wallpaper group theories resulted in classification based on transformation arrangements (translation, rotation, and reflection). This has led us to develop software to classify IGP designs based on normalization grid methodology.

Problem 2: The fundamental principles of all IGP design methodologies may vary based on the experiences and disposition of the designer. This has led us to develop software to explore the IGP based on the design motif.

A variety of computer applications have been developed to redesign the IGP, but little has been done in their classification and naming convention. The research attempt is to develop an ambitious software tool using (C#) in the (.NET) platform.

This thesis will state the two main problems with a clear objective of producing an accurate system grid. This combined with the need for specialized knowledge about the mathematical rules and algorithms to output classification and designs of IGP.

In recent years, we have seen a revived interest in IGP designs of what was once a neglected area of research [5]. The approach of this thesis based on earliest civilizations, cultures, and different religions from earlier than 3000 B.C. Architects, astrologists, philosophers, and mathematicians showed great interest in understanding the IGP reflected from scientific/humanities relevance to their effect on the design of religious buildings, to the healing of some illnesses and development of mathematics.

Computer modeling provides visual expression and new motivation in algorithmic presentation for classification and design of IGP, while the underlined messages of sacred meanings remain intact.

The purpose of this research is to investigate the possibilities of using software to supplement human power in accomplishing the tasks related to the classification and design of IGP. Its main hypothesis is that this software offers many advantages over human power in terms of recognizing the system grid for producing accurate design and classification digitally for further variety. It also has a great capacity for testing many designs quickly, accurately, and easily compared to the limitations of the human's capabilities. Although this research focuses specifically on IGP classification and design, the underlying goal is to develop the general ideas and principles that be applied to other styles.

The mathematical aspect of IGP has received extensive attention and the detailed work of Frithjof Schuon [6] and Titus Burckhardt [7] has always been a source of inspiration and guidance for many researchers. Bourgoin published his analysis in France in 1879 [8]. The computer modeling provided a new motivation in their algorithmic representation for further exploration of their application in modern architectural design and their symbolic meaning [9]. The established techniques of G. E. Martin on transformation geometry [10], T. Pavlidis on algorithms for graphics and image processing [11], Doris Schattschneider on the plane symmetry groups and their recognition and notation [12], the writings of K. Critchlow that analyses the geometry of IGP from a metaphysical and cosmological point of view [13], the concerns with the spiritual dimension of their symbolism [14] mathematics of their design [15], and aesthetic aspects of their spatial and tonal domains [16]. These works have had a profound influence on a group of young architects and art historians [17], which is likely to provide a new framework for design expression.

The successful outcome of such representation would not only offer a rich resource for art communities but also would be of interest to mathematicians, crystallographers, architects, and others. They can serve as elegant test beds for research into hierarchical programming and texture mapping.

This research focuses on exploring IGP designs and their physical and formal qualities. There have been many research studies, which have investigated classification of IGP within the context of 7-frize and 17-wallpaper group theories and not based on their grid system. The precedent for researching classification and design has been to use normalization grid system methodology within a given mathematical frame for the adoption of innovation to occur within a software.

Upon the observations of many research studies, noted that any research into IGP designs is essential to combine the scientific/humanities relevance and its relevance to beneficiaries. This research acknowledges that IGP are the products of Islamic culture and have significance within the scope of sacred meanings. This research study is based on the stand not to be influenced by any position and the credibility is not compromised by fact that the researcher comes from the same culture that produced such art and aims to remain objective.

The research subject is based only on absolute knowledge and concerning classification and design. The inner core of this research is defined by how successful the software is in performing IGP classification and design. The outer core of this research is concerned with the generalization of results and the applicability of the software to design and classify complex IGP. Given the span of material for accounting this framework is interdisciplinary, computer graphics, programming, psychology, geography, and sacred. The concept is foundational to normalization grid methodology seen in that angle in the study. In this thesis, it has been given the outline of how systematically investigated the IGP to formulate a more thorough understanding of its grid system. The features and framework of IGP, defining what an IGP is and discussing the dimensions of the relationship between its designs and sacred meanings discussed.

In conclusion, a discussion of the practical limitations encountered during this research and the implications of IGP normalization grid methodology theory is given. Given the spatial nature of the computer graphics format and the mathematical structure of IGP, it is possible to incorporate the two logical systems within the computer environment to develop a software based on normalization grid methodology capable of classifying, identifying the motif the will led to the IGP exploration. This research addresses the question of how to integrate the mathematical and algorithmic rules of with visual environment techniques using computer tools to develop a software capable of reading the design in terms of motif for classification and further design exploration based on normalization grid methodology. The scope of this thesis varies based on the individual chapters.

Chapter 1: Introduction
The researcher lays the groundwork for the classification and design of IGP, discussing the statement of the problem, theoretical framework, hypothesis, rationale, purpose, and significance of the research.

Chapter 2: Literature Review
The researcher reviews related literature to share with the reader results of other studies that are closely related to the study that is being reported, comparing the results of the study with other findings, establishing the importance of the study, so that the investigator can choose the right way to proceed with the envision objectives.

Chapter 3: Background
The researcher discusses the revived interest in sacred meanings behind IGP in pre-modern Islam from the Sufi perspective and modern contradictions in design philosophy. The scope extended to the limits of the religious philosophy of Islam. Since the philosophy of Islam does not have any boundaries, it was immensely difficult for us to view the scope of this chapter through a limited prism. The task of deciphering the hidden meanings and the subtle truths contributed to an immense enrichment of Islamic knowledge. Also, in the same chapter, the researcher reviews the classification and design based on the grid system led to the study of the existing classification systems and works of eminent experts like X-Lee, Glassner, Abas, and Salman, etc. The fact that we are concluding the work of the above experts complicated the magnitude of our task since we must take a holistic view of the entire spectrum of works that existed on this subject. However, the task of presenting a new method of classification and design rightly demanded a very scientifically validated method through incorporating algorithms and mathematical rules into adapted IGP normalization grid methodology to develop a software that proves the validity of such a line of research.

Chapter 4: Program Design
The researcher discusses the software design analysis as the preliminary step, which is also a building block of software engineering. Nevertheless, the researcher discusses the existing and proposed classification and design systems. The study will decide if the proposed system will be cost-effective from the research point of view and if can be developed in the given existing budgetary constraints. The chapter shows the Data Flow Diagram (DFD), which is a network that describes the flow of data and processes that change or transform data throughout the system.

Chapter 5: Program Implementation and Experimental Results
The chapter discusses the conceptual design phase, which is the initial phase of research and involves the intellectual process of developing the basic functions, operational characteristics, and the overall organization of tasks. The researcher tried to build something on the windows platform since it was more widely used and we were familiar with it, however, in the future we would like to build a more powerful package using the latest (.net) technologies. The attempt is to develop professional software; rather, it explores an idealized problem whose solution might lead to advances in developing professional tools.

Chapter 6: Conclusion and Future Works
The chapter deals with the conclusion, limitations, recommendations, and future scope for research.

Chapter 2: Literature Review

Before launching research, it is essential for the researcher to survey the studies made earlier, which are significant to the topic on hand. Proper understanding of the problem and the procedure is the pre-requisite for any successful research. Such an understanding will be possible only when the researcher gets familiarized with the related literature. The review of the research materials in a research study serves several purposes. It shares with the reader results of other studies that are closely related. It relates the study to the larger, ongoing dialogue on the topic, filling in gaps, and extending prior studies. It provides a framework for establishing the importance of the study as well as a benchmark for comparing the results of the study with other findings. Considering the early research, the problems can be viewed from different perspectives so that the investigator can choose the right way to proceed with the envision objectives.

The body of literature related to this thesis covers two main areas. The first area includes studies related to its origin, spirituality, symbolism, symmetry, mathematical rules, pattern group theories, classification systems, and design methodologies. The second area is the potential use of software in IGP classification and design. The majority of the patterns subjected to the study appear in the books of Bourgoin [8], Critchlow [13], El-Said, and Parman [18], and David Wade [19]. The patterns and symmetries are studied using the CAD package archived to make it possible to recreate IGP accurately.

Traditionalists and Art Historians

The traditionalists wanted to reintroduce into the life of modern Western society the long-forgotten spiritual essence and traditional wisdom. Western civilization since the European Renaissance might have advanced in natural sciences but declined in the spiritual aspects destroying Eastern traditions in the name of modernity and progress. The traditionalists approach the question of creative art from this viewpoint and of religious inspiration. The traditionalists approach the past for useful renewal of traditional wisdom seeking spiritual fulfilment. The founder of the traditionalist approach was the French metaphysician Rene Guenon [20], who devoted his life to the study and revival of traditional sciences through Hinduism and Islam. His ideas influenced the metaphysician and art historian Ananda Coomaraswamy [21] and Swiss metaphysician Frithjof Schuon [6] who developed a different approach to the study of symbolism and metaphysics that attracted many followers, are contributed and shared their passion for tradition such as Titus Burckhardt [7], Seyyed Hossein Nasr [5], and Martin Lings [22] for they have extended the traditionalist approach to the study of premodern Islamic art and architectural heritage. The traditionalist approach has inspired several studies on Islamic art and architecture as done by Nader Ardalan and Laleh Bakhtiar [23], Critchlow [13], Adrian Snodgrass and Richard Coyne [24]. Their efforts were complemented by the writings of creative scholars such as Annemarie Schimmel [25], Henry Corbin [26], Louis Massignon and Benjamin Clark [27], Toshihiko Izutsu [28], and Hellmut Ritter [29], who made significant contributions to the interpretation and understanding of medieval mystical sciences in general and that of Sufism in particular. The approach of symbolism has also benefited much from the work of anthropologist and historian of religion Mircea Eliade and Philip Mairet [30]. Their works have opened a new horizon of understanding and provided with new intellectual tools for rethinking the field's long-established art and architecture. Through symbolism, they establish a means to interpret the human conditions of existence in cosmological terms.

The art historians approach the past for objective reconstruction of the historical reality, seeking to satisfy academic curiosity and a desire for knowledge. This significant difference extends beyond methodology, making it difficult to measure the rigor of both approaches against the same criteria. A fundamental difference between the two approaches lies in the purpose and usefulness of each undertaking. The question that lies at the heart of both approaches seems to be: what is the legitimate mode of capturing this all-unifying adjective, history or symbolism, culture, or spirituality? In their attempts to explain the difference that distinguishes art and architecture of premodern Islam, both the traditionalists and the art historians reveal a continuous struggle with this deep-seated orientalist preoccupation [31].

Horror Vacui

In general, one aspect of decorations has been associated with horror vacui (literally: fear of empty spaces). In physics, horror vacui is a theory first proposed by Aristotle which demonstrates that nature abhors a vacuum, and therefore space would always be trying to suck in gas or liquids to avoid being empty. In visual art, horror vacui is the filling of artwork with detail. The term is associated with the Italian critic and scholar Mario Praz [32], who used it to describe the suffocating atmosphere and clutter of interior design in the Victorian age. The arrangement of Ancient Egyptian hieroglyphs suggests an abhorrence of space. Signs are repeated or phonetic complements added to prevent gaps. Another example comes from ancient Greece during the Geometric Age (1100 - 900 BCE) when horror vacui was considered a stylistic element of all art. In architecture, this term has been used to characterize the human desire to decorate every surface of a building with pattern and texture. Sir Ernst Gombrich [33] is the author of the book The Story of Art, in which he uses the term "cult of restraint" to refer to those who reject decoration because of its superficiality, and praise objects that convey their essence without the need to advertise it via decoration. Architects like Mies van der Rohe [34] and Le Corbusier [35], as well as Walter Gropius [36] and the Italian Futurists such as Filippo Tommaso Marinetti [37], were pioneers who rebelled against overuse of decoration (cult of restraint) and revelled in the beauty of technology and machines that promised to change the world for the better. On the other hand, some artists such as Robert Jensen [38] and Patricia Conway [39] made a return to horror vacui in the form of deco-rationalism and they were among the Pattern and Decoration art movement (P&D) situated in the United States from the mid-1970 to the early 1980s. There were also connections between the Pattern and Decoration movement and the Feminist art movement. Overall, it seems as if the forces of modernism and deco-rationalism are both active in contemporary architecture.

History of Islamic Art

The Oxford English Dictionary defines the word "design" as an arrangement of lines, shapes, or figures as decoration; a pattern: floral/abstract designs. The same dictionary interprets the word "language" as signs, symbols, gestures, etc used to indicate ideas or feelings [40]. Islamic art encompasses great achievements in geometry, calligraphy, and arabesque. These visually diverse art forms grow out from the same spiritual origin to represent the multiple manifestations of the divine [13] [41]. Symmetric Islamic patterns come in three distinct geometrical flavours.

1. **Calligraphy**
   Most often mold Arabic lettering for words such as Allah and Mohammed or short verses from the Quran, to create symmetric geometrical forms.

2. **Arabesque**
   In such patterns, spiral forms intertwine, undulate, and coalesce rhythmically to produce stylized leaves and floral forms.

3. **Islamic Geometric Patterns IGP**
   Employs polygons, and less frequently, regions bounded by circular arcs, to produce space-filling patterns. The researcher here analyses only IGP designs based on its symmetry and grid system.

When such patterns are rendered on a two-dimensional flat surface, a basic unit cell repeats itself repeatedly. A consequence of this is that there is no natural point of focus for the eye. As one looks at an expanse of pattern the eye 'flows' continuously following the lines and seeing a variety of intricate structures and relationships [45]. The tradition was completely inspired by a deep religious philosophical and cosmological approach, which embodied all aspects of life and manifested itself in every product [13] [41]. History of art, being a modem science inevitably approaches Islamic art in the purely analytical way of all modern sciences, by dissection and reduction to historical circumstances [7] [42] [43]. For more than thirteen centuries they acted as unifying factors. They have linked the architectural products from all over the Islamic world, extending across Europe, Africa, and Asia. The four fundamental concepts in Islam: beauty, harmony, symmetry, and unity are all intrinsic to the contemplative side of Islamic Art [44]. Geometric patterns occur in rich profusion throughout Islamic cultures. They are found on diversity of materials – tiles, bricks, wood, brass, paper, plaster, glass, and on many types of objects. They occur on carpets, windows, doors, screens, railings, bowls, furniture-specially pulpits in mosques, and on other surfaces. They can be seen in abundance in Persian miniatures in the illumination of the Holy Koran and on architectural surfaces of mosques, palaces, madrasas (centres of education), and tombs [45]. Works of art and in general the visually perceived environment have extraordinary power in shaping the lives and thoughts of men and women and that ornament is the ultimate mediator, paradoxically questioning the value of meanings by channelling them into pleasure [46].

Origins and Evolution

Consider the year 622 (the year of the emigration of the followers of Mohammed from Mecca to Medina) as the beginning of Islam [47]. The rise of Islamic culture in the seventh century has marked the beginning of a new artistic, decorative, and sacred tradition [48]. El-Said portrays a historical dimension to the origin of IGP and its developments. He noted that the early mastery of Arab's knowledge of mathematics, rhythmic repetitions and symmetries gained from Byzantium and Sassanid era is the reason for the quick expansion of such motifs after Islam [49]. We find that within a relatively short period-about 100 years the Muslims conquered the Arabic peninsula, North Africa, Spain, Southern France (for a while), Sicily, the whole territory called the Middle East today and penetrated Central Asia and India. Several significant cultures were flourishing in these areas at the time, and their effects on the Muslim world and thought gave birth to such works of art …Their quick expansion brought them into conflict with the two great political powers of the time. Byzantium and Sassanid Iran, both having a well-developed, flourishing culture…Muslim arts were influenced and inspired by both [47].

The use of IGP is one of the chief characteristics that give the Islamic artistic heritage its distinct identity. Geometry as an abstract art form was developed in part due to the discouragement of images in Islam on the basis that it could lead to idolatry [50]. Rozsa revealed in her study that when one enters a mosque, one does not see paintings or sculptures, but instead finds decorations of various kind of carving along the walls, which are usually referred to as "geometric". The reason for this is not the inability of Muslim artists to describe living beings but the holy book of Islam, the Quran, forbids the description of beings having a soul … "*God being One and having no companion, it is only He, who has the power and ability to create the perfection of which man can never reach*". "*Al-musuwwir*" (Bestower of Forms), one of His 99 attributes, is derived from the same root as the word "picture" (*Surah*). In the Arabic translation of the Old Testament, this very word is used to denote the subject of prohibition, proving the relations and the interconnections of the great monotheistic religions referred to above" [47].

Grabar said, "*If the authors are right in their analysis, it's quite extraordinary, I always knew buildings from this era were different, but I never knew the difference was in the nature of their geometry*" [46]. Complex geometry in the art of the period trickled down from the achievements of mathematical scholars. Muslims in Baghdad had made the first translations of Euclid's geometry in the 9th Century, giving them a 200-year head start on European intellectuals and architects [51]. Joshua Socolar professor of physics at Duke University said that Physicists were excited about this stuff in late 1980 … "*It's fascinating that you would find these patterns in such old architecture*" [52].

Glazed ceramic tiles decorated with intricate IGP and Arabic writing were for centuries, and still is, in widespread use in the Islamic countries and for the westerners, it remains as one of the most recognizable and constant marks of the beauty of mosques. From its origin in the Middle East and flourishing in the Islamic world, glazed tiles spread to Spain and Portugal and subsequently to Italy, the Low Countries, and most of Europe. Glazed ceramic tiles are used as decorative panels or as finishes of whole walls and façades in many countries, such as Portugal, where they make up a very relevant part of the national cultural heritage. Introduced to the country from the Moorish and Andalusia factories of Granada, Seville, and Manises, they started being produced locally in the 16^{th} century and developed as a preferred means of finishing architectural surfaces a century later, when they became ubiquitous in palaces, churches, gardens, and bourgeois houses. [52]. A new study shows that Muslim artisans working in what is now Iran went further, producing a sophisticated geometric pattern more than 500 years before Western mathematicians identified it in the 1970 and 1980 [53]. Oleg Grabar said Islamic architects started using geometry to express the holy … "*It suggested something transcendental and extraordinary*." [46].

Symbolism

Symbolism has long been associated with icons. Both are visual and present a powerful insight into something unknown or underneath. According to the dictionary, an icon is either: an image; esp., a religious image pointed on a wood panel, or a small picture on a computer display that suggests the purpose of an available function [40].

God has communicated to mankind through the language of icons [54]. However, looking around us we see icons everywhere; the sun is an icon for energy, water is an icon for life, a book is an icon for knowledge, a synagogue, church, or a mosque are icons for the house of God. Icons have been utilized by all major religions, cultures, communities, cults, faiths, civilizations to represent in a very effective manner their beliefs. A religious symbol conveys its message even if it is no longer consciously understood in every part. For a symbol speaks to the whole human being and not only to the intelligence [Mircea Eliade]. In the Jewish faith, the Chanukah Menorah is an icon of miracle, and hope. The cross is not an icon for worship, and whoever does would be guilty of breaking the second commandment. The Cross-is a poignant symbol of the ordeal Jesus Christ. Another form of sacred icons is the ancient Egyptian hieroglyphics form of writing in which pictorial symbols are used to represent ideas and sounds. They used such writing mainly for religious inscriptions on temples and stone monuments and to record the words and deeds of royalty. The Egyptians sometimes called their writing the words of God [55].

Each context of a symbol reveals something more which was only unformed and allusive in the neighbouring contexts. In this sense, symbols imbue human existence with significance by pointing to a more profound, more mysterious side of life, to the miraculous and sacramental dimensions of human existence. For this reason, symbols related to ultimate reality are viewed as a source of inspiration and revelation [30]. Grabar discusses the methodological problems of the current discourses on symbolism, identifying three inherent shortcomings [46]:

1. **Absence of Scientific Precision**
   That is, the lack of an explicit link between historical data and mystical interpretations of built forms.

2. **The Ambiguity of the Islamic Character**
   That is considered unique but "never described."

3. **Absence of the Contemporary Context**
   That grounds interpretations in existing literary documents, which would prevent the unavoidable impression of modern constructs, perhaps valid to modern man, applied to traditional forms.

Such a revitalization of the traditional sciences, however, requires a re-discovery of the true meaning of symbolism and the education of modern man to understand the language of symbolism in the same way that he is taught to master the languages of logic or mathematics [56]. The meanings of symbols are not intentionally constructed but rather discovered or revealed through reflections on transcendental realities, and consequently, the efficacy of a symbol does not depend on its being understood [30].

Symbology has always been understood as a means of communication and played a role in humanity's development. Symbolism is only there to facilitate and represent ideas and emotions. Symbols take many forms and they each have their energy such as shapes, colours, and words. Signs refer to material objects whereas symbols refer to immaterial concepts. However, looking around us we see symbols everywhere; the sun is a symbol for energy, water is a symbol for life, a book is a symbol for knowledge, a synagogue, church, or a mosque are symbols for the house of God. Symbolism is a current topic in many disciplines; psychologists, anthropologists, cultural theorists, social scientists, historians of religion, historians of art and architecture, philosophers, and architectural theorists, have all developed viable dimensions of the discourse. Symbols have been utilized by all major religions, cultures, communities, cults, faiths, civilizations to represent in a very effective manner their beliefs. Symbolism is presented as the language of religion that divinity speaks. The traditionalists viewed tradition and religion to be the main elements characterizing premodern society, be it Islamic, Christian, Hebrew, Buddhist, or Hindu. It is religion, they argue, that provides a community with the divine principles, and it is a tradition that weaves these principles into peoples' modes of living, thinking, and making. The art historians approach the past for objective reconstruction of the historical reality, seeking to satisfy academic curiosity and a desire for knowledge. Ernst Gombrich provides a thorough account of the history of writings on art decoration [33]. Max Weber and Ephraim Fischoff described religion as a system of sacred religious symbolism [126]. Carl Jung emphasized the importance of balance and harmony. He cautioned that modern people rely too heavily on science and logic and would benefit from integrating spirituality and appreciation of unconscious realms [127]. The anthropologist Clifford Geertz defined religion as a system of symbols [128]. Edward Evans-Pritchard did not share the hope that a theory of symbolism and religion could ever be found [129]. In different civilizations, symbols were and are visible in their sacred beliefs [43].

1. **The Egyptian Pyramids**
   Thought to symbolize the primordial mound from which the Egyptians believed the earth was created and the descending rays of the sun.

2. **The Egyptian Hieroglyphic**
   The word hieroglyphics comes from two Greek words that mean sacred carving in which pictorial symbols are used to represent ideas and sounds. They used such writing mainly for religious inscriptions on temples and stone monuments.

3. **The Sumerian Pentagrams**:
   It is found in Mesopotamian writings to symbolize corner, angle, cavity, and hole. The edges; forward, backward, left, right, and above had an astrological meaning, symbolizing the five planets Jupiter, Mercury, Mars and Saturn, and Venus as the Queen of Heaven (Ishtar).

4. **The Hindu Ouroboros**
   The serpent devouring and eating itself symbolizes the cyclical nature of the universe: creation out of destruction, life out of death, the concept of infinity, the eternal cycle of renewal, and the universe without boundaries, limits, or end.

5. **The Hawaiian Huna**
   Tad James illustrates the power of Hawaiian Huna in meditation with geometric symbols (circle, triangle, and square) in a certain order caused a shift in consciousness, as well as dramatic openings in the energy circuitry of the body [130].

6. **The Roman Divine Ratio**
   The ancient Roman architect Marcus Vitruvius explains that builders should always use precise ratios when constructing temples. For without symmetry and proportion no temple can have a regular plan. Scientists and philosophers later discovered that the same ratio Vitruvius was seen in the human body - 1 to PHI (1.618) - exists in every part of nature, from swimming fish to swirling planets. This divine ratio or divine proportion has been symbolizing the building block of all life.

7. **The Jewish Chanukah Menorah**
   It is symbolizing the light for freedom, tolerance, peace, courage, knowledge, charity, responsibility, and continuity.

8. **The Christian Cross**
   According to the Christian second commandment (Exodus 20:4) the cross is not an icon for worship, and whoever does would be guilty of breaking the second commandment. The Cross is a poignant symbol of the ordeal Jesus Christ underwent to save humankind from their sins.

9. **The Islamic Geometric Pattern IGP and Calligraphy**
   Muslims used IGP to decorate their buildings of worship. The mosque is the most important building for Muslims, a place of religious instruction and temporary homes for traveling scholars. In numerous mosques, flat surfaces, domes, and minarets are decorated with sacred scriptural quotations of the Quran in elegant calligraphy.

Symbolism is understood to be based on two domains of reality; the physical representing the spiritual and of two fundamentally different kinds; Universal symbols relate to their innate nature, such as geometrical or numerical symbols, whereas symbols relate to a particular tradition such as the cross as a symbol of resurrection in Christianity and alphabetical symbolism in Islam. The truth beyond the meaning of the symbolism is beyond culture and language. Meanings need to find expression, a vehicle to express itself. In the case of humanity, that vehicle is language. When we speak, we can do so in thought, which is beyond language, and yet to communicate we choose a language.

Symmetry and Asymmetry

The original conception of symmetry, as conveyed by the dictionary is expressed with words such as beauty'' balance, and harmony. The word was and still is used to refer to a balance of components as a whole [40]. The contemporary non-scientific usage of the word refers to an object whose left and right halves correspond through the reflection in a mirror. Thus, a human figure, or a balance scale measuring equal weights, may be said to possess symmetry [64]. Symmetry means a balance, a repetition of parts, or simple uniformity of form. Symmetry simply means pattern. But the range of symmetry is far more than simply appealing architecture and pretty patterns. Hermann in his book discussed the possible link between symmetry and beauty. Beauty is bound up with symmetry. Symmetric means something like well proportioned, well balanced, and symmetry denotes that sort of concordance of several parts by which they integrate into a whole [64]. In addition to its usefulness in many branches of science, symmetry is often used to study art and ornament [65] [66]. Symmetry can be described in exact terms of the canons or preferences exhibited in the visual (primary ornamental) art created by a given person or school [67]. Certainly, it is not difficult to see how symmetries of various forms, be they in the natural world or the artificial world of human aesthetics, are credited with beauty: the reflection of a mountain in a lake, a starfish, flowers of many types, a honeycomb, snowflakes, the symmetry of a face, the facade of a cathedral, a Byzantine mosaic of Christ Pantocrator in a Greek church – the list could be endless [68]. Symmetry generally conveys two primary meanings. The first is an imprecise sense of harmonious or aesthetically pleasing proportionality and balance such that it reflects beauty or perfection. The second meaning is a precise and well-defined concept of balance or "patterned self-similarity" that can be demonstrated or proved according to the rules of a formal system: by geometry, through physics or otherwise. Although the meanings are distinguishable in some contexts, both meanings of "symmetry" are related and discussed in parallel [64].

M.C. Escher interacted with the growth of symmetry theory, creating new art based on the mathematical results that emerged during his lifetime [69]. However, mathematically symmetry can be simply defined in terms of invariance of properties of sets under transformation [70]. The mathematical tools behind a formal treatment of symmetry are relatively new, but our appreciation of symmetric patterns goes back millennia [64]. Symmetry and beauty are often claimed to be linked, particularly by mathematicians and scientists. However, philosophers and art historians seem generally to agree that although symmetry is indeed attractive, there is also a somewhat sterile rigidity about it, which can make it less attractive than the more dynamic, less predictable beauty associated with asymmetry [68]. Many artists of modern time interacted with the growth of symmetry theory, creating new art based on the mathematical results that emerged during their lifetimes [71].

Symmetry is a wonderful theoretical concept for science, providing structure, organization, and simplification for a host of complex, apparently unrelated phenomena across many disciplines. However, seductive though symmetry is as a concept, there is much evidence that not only is asymmetry found in the sub-atomic world of physics, and throughout the biological world at all levels, from biochemicals to brains, but that asymmetry is also exploited and developed in the art as well. Symmetry, although mathematically fascinating, also has coldness, a rigidity, and fixity, which is less interesting, less attractive, and indeed less beautiful than asymmetry. Too much asymmetry is, however, mere chaos. Asymmetry, when it is used in the arts, is used to season symmetry. However, without an understanding of the deep mathematical structures of symmetry, we would not be able to realize how asymmetry is generated. Symmetry and asymmetry are therefore an essential dialectic for both science and aesthetics [68]. Arnheim has also argued that there is an underlying cognitive scale beneath the dimension of symmetric–asymmetric, which corresponds to simplicity– complexity [117]. Weyl recognized this tension and described how 'occidental art, like life itself, is inclined to mitigate, to loosen, to modify, even to break strict symmetry [64]. Pure symmetry is somehow too harsh, too rigid, and un-life like, was suggested by Immanuel Kant, who commented on how, all stiff regularity (such as borders on mathematical regularity) is inherently repugnant to taste, in that the contemplation of it affords us no lasting entertainment… and we get heartily tired of it [72]. The art historian, Ernst Gombrich was of a similar mind, seeing a banality within symmetry: Once we have grasped the principle of order, we can learn the thing by heart. We have easily seen enough of it because it holds no more surprise, so that, symmetry and asymmetry are seen as, a struggle between two opponents of equal power, the formless chaos, on which we impose our ideas, and the all too formed monotony, which we brighten up by new accents [33]. Although a little asymmetry can be beautiful, an excess merely results in chaos [68]. As Adorno suggested, asymmetry probably results most effectively in beauty when the underlying symmetry upon which it is built is still apparent [73].

Symmetry-Asymmetry and Human Brain

Andreas Speiser required that all geometric pattern designs have some degree of symmetry [81]. Symmetry can even be identified by the thought process of the human brain. There is also a significant neurological and psychological basis for the appreciation of symmetry. Christopher W. Tyler used a computer to experiment with functional brain imagining showed that the human eye is particularly fast and accurate in the detection of objects with vertical mirror symmetry. Once the eye detects a line of vertical mirror symmetry, it goes on to explore only one half of the scene, the other half taken as understood [125]. Edmund S. Howe explains that subjective ratings of goodness correlated precisely with the degree of symmetry present [76]. Anders Pape Moller and John P. Swaddle explain their theory that asymmetry is related to genetic stability and fitness, and that symmetric individuals appear to have quantifiable and significant advantages over their asymmetric counterparts [78]. A. V. Shubnikov discusses the psychological effects of specific wallpaper groups where decoration with those symmetries might be most appropriate [65]. The range of symmetry is far more than simply appealing and pretty patterns. According to Rosen symmetry usually means a balance, a repetition of parts, a regularity of form. More precisely and generally symmetry can be said to be invariance under transformation [47]. This evidence provides us explanation for the importance of symmetry and asymmetry in art with a human connection.

The Psychology behind Symmetry

There is a significant neurological and psychological basis for our appreciation of symmetry. The science of psycho-aesthetics attempts to quantify our aesthetic response to sensory input. Research in psych aesthetics shows that our aesthetic judgment of a visual stimulus derives from the arousal created and sustained by the experience of exploring and assimilating the stimulus [104]. Experiments with functional brain imagining show that humans can accurately discern symmetric objects in less than one-twentieth of a second [74]. By tracking eye fixations during the viewing of symmetry the eye will explore only non-redundant parts of that scene. Once the eye detects a line of vertical mirror symmetry, it goes on to explore only one half of the scene, the other half taken as understood [75]. Subjective ratings of goodness correlated precisely with the degree of symmetry present [76]. In a similar domain has been found that subjects preferred to arrange points symmetrically in a grid [77]. Others simply state that humans find symmetrical objects more aesthetically pleasing than asymmetric objects [78]. From the arousal point of view, the experiment has shown that an increase in symmetry is met with a reduction in arousal [75]. The aesthetic effects resulting from the symmetry lies in the psychic process associated with the discovery of its laws [65]. But these experiments and theories reveal that we do have some hard-wired reaction to symmetry, a reaction that affects our perception of the world. This evidence provides us with a partial explanation for the historical importance of symmetry in ornament, and some confidence in its continued aesthetic value [31].

Design Methodologies

Relatively recently, starting with the publication of the pioneering work of Bourgoin in 1879, several authors have published a large collection of Islamic patterns. They offered their analysis of the methods of constructions [86]. It's unclear whether artists of the time have fully understood the unique features of their patterns [7]. A conceptual breakthrough occurred by 1200 A. D., in which girih patterns were reconceived as tessellations of a special set of equilateral polygons (girih tiles) decorated with lines. These girih tiles enabled the creation of increasingly complex periodic girih patterns, and by the 15th century, the tessellation approach was combined with self-similar transformations to construct nearly perfect quasicrystal line patterns. These patterns have remarkable properties; they do not repeat periodically and have special symmetry—and were not understood in the West until the 1970. They also discussed some of the properties of Islamic quasicrystal line tiling, and their relation to the Penrose tiling, perhaps the best-known quasicrystal pattern [90].

Kaplan in his article reported throughout history IGP have formed an important part of art and ornamental design. He also presented our unprecedented ability to understand ornamental styles of the past, to recreate traditional designs, and to innovate with new interpretations of old styles and with new styles altogether [31]. Abas and Salman's methods of constructions study them in the context of tiling with a simple view that these are a set of tessellation primitives such as triangles, rectangles, squares, hexagons, and octagons. Elsewhere, they argue for a simple approach tied to the tools available to designers of the time [84]. Another method is of reflecting lines of periodically placed circles [87]. Also, there is a technique based on the construction of networks of eightfold stars [88]. The decomposition of periodic Islamic patterns by their symmetry groups, obtain a fundamental region they use to derive properties of the original pattern [2]. The overall impression that is created is that from the earliest of times the inventors of Islamic patterns were dedicated geometers inspired by theoretical compass/ruler-based construction of the classical Greek geometry [86].

Islamic architecture establishes a recognizable mathematical order and has a special concern for aesthetics. One complements the other. This is exemplified by taking an analytical study and in tracing out certain unknown facts about the design of unquestionably the most ambitious architectural project such as the Taj Mahal, the most notable building achievement of the Mughal Empire [89]. The decagonal patterns on the Darb-i Imam shrine (Isfahan, Iran, 1453 C.E.) are quasi-periodic and were constructed by tessellation, using a set of five tile types, which they called girih tiles [53]. Architects did use, a swift technique to design and implement the decagonal patterns based on three types of overlapped but hidden grids with the same interval (two of them are tilted on both sides by 72° and 36°), using only the T-square and two triangles: 18°/72° and 54°/36°, without using the slow compass [88]. If one asks the question as to how the Islamic patterns, or patterns of any culture, originated, then it would seem most logical to start with the practical experience of tiling and covering with simple naturally occurring shapes. The shapes would then be worked on giving rise to triangles, rectangles, squares, hexagons, and circles. The shapes would have been decorated with simple colours and patterns. From this beginning, the next stage would be to experiment with multiple shaped tiles, produced by overlapping tiles, and to invent more pleasing decorations [86]. Before preserving these delicate artifacts, it is important to produce accurate virtual reconstructions that will help professionals to make decisions and to test the different options [91].

Design Characteristics

It has been included to make the point that IGP occur in many shapes and some rather simple ones do not possess the instantly recognizable Islamic flavour, which is displayed by the vast bulk of the more complex patterns. The most striking characteristic of IGP is the prominence of star and rosette shapes. Such shapes with five, six, eight, ten, twelve, and sixteen rays are the ones that occur most frequently, but patterns containing other numbers, particularly in multiples of eight up to ninety-six, can be found [45].

Design Properties

The vast variety of geometric forms and the strict rules of its generation reveal an important inner dimension of Islamic tradition: "*unity in multiplicity and multiplicity in unity*" [42]. This principle is represented using different mathematical forms symbolizing the constant celestial archetypes within the cosmos [12]. Islamic geometry has successfully integrated the bounding laws of pattern geometry with the beauty and harmony of colours and rich combinations [13]. Although these patterns are physically fixed in time and space, still their visual manipulation and rhythm indicate movements in these dimensions [50]. The complexity of Islamic patterns is stressed using different colours, reflecting, and shining materials and glazes, the contrasting of textures and the manipulation of planes Visual effects are based on these properties [42]. Most of these IGP properties can be grouped under the following categories:

1. **Root Two**
   IGP is based on the square repeat unit and the "root two system" of proportion. The constructions of the square repeat unit and the division of the circle into four, or multiples of four, equal parts [13] [49].

2. **Root Three**
   IGP based on the hexagonal repeat unit and the "root three system" of proportion. The constructions of the hexagonal repeat unit and the division of the circle into three, or multiples of three, equal parts [13] [49].

3. **Golden Ratio**
   IGP based on the pentagon repeat unit and the "golden ratio" proportion system. This includes all patterns generated by the division of the circle into five and all patterns generated from the multiples of five [13] [50].

4. **Unit Pattern Repetition**
   IGP is made up of a single repeated geometric unit; through the principles of repetition and symmetry [31] [92]. The making of a complex whole by the repetition of basic elements satisfies both an apparent need for intense visual stimulation and the Islamic search for unity [13] [49].

5. **Two-Dimensionality**
   IGP maintains two-dimensionality by fitting all the polygonal shapes together like the pieces of a puzzle, leaving no gaps. Such a construction is known mathematically as a "tessellation" [93].

6. **Symmetrical**
   Any IGP polygon in a design can be chosen as a central point from which the pattern radiates symmetrically [31] [92][94]. Some patterns are radiating from a star surrounded by different polygons [42].

7. **Expansion**
   IGP are not designed to fit within a frame. The expansion forms the basis of the characteristic of the IGP. Since the patterns consist of repeating elements that move outward from a central point, they could go on indefinitely but are limited by the edges of the space they decorate [95].

8. **Basic Grids**
   They are constructed based on a limited number of basic grids, which are generated from patterns of circles [96]. The complex geometric patterns are all elaborations of simpler constructions of circles, which are often used to determine grids. Mathematically these grids are known as regular tessellations, in which one regular polygon is repeated to fill the plane [13] [17]. The main basic grids are:

a) basic grids are based on equilateral triangles and hexagons and their multiples.
b) basic grids are based on the squares, octagons, and their multiples.
c) basic grids based on the pentagon and its multiples.
d) basic grids are based on a combination of basic categories.

Even though IGP are generated from simple forms; they were combined, duplicated, interlaced, and arranged in fascinating combinations, which became one of the most distinguishing features of Islamic art. Although they are generated based on very strict rules of geometry, the geometric ornamentation suggests a remarkable amount of freedom, in its repetition and complexity. It offers the possibility of infinite growth and can accommodate the incorporation of other types of ornamentation as well [42].

9. **Color**
The use of color is an important characterization of Islamic decoration, which was supported by great skill in mixing and producing colors. By applying these colors to different designs, a new relationship is developed based on the established rules of density, succession, and contrast [97]. Generally, in Islamic art, certain colors are used more often than others. The choice of colors should reinforce the effect of the decorative composition, which creates special effects intended to highlight the optical effects of the geometrical compositions [42]. Red, orange, and gold are associated with fire and the sun. Therefore, they are used to suggest warmth and heat. Green and blue are associated with cold lunar developments and are used to suggest cold. If colors from different groups are applied, the effect will produce a sense of dynamics and amazing visual pleasures [97].

10. **Texture**
The application of patterns in different materials added a rich diversity of textures, which contributed to the fascinating optical effects. Even though different materials were used, they were unified by geometric principles [42]. These patterns were applied to stonework, metalwork, woodwork, ceramics, textiles, carpets, miniatures, and architectural decoration [95] [17].

11. **Contrast**
This effect was achieved by playing with negative and positive areas. These were created either by using different contrast colors, by curved patterns, which permits the play of light and shade, or by using different surface textures and different reflective materials [42] [97].

Design Construction and Formation

The act of designing these patterns was considered a form of worship and encapsulated a divine religious experience. These artists and the methods they used were secretive, and only a few passed on this tradition until it was lost [84]. The generating force of patterns lies in the centre of the circle, which represents the point at which all Islamic patterns begin. It is the symbol of a religion that emphasizes one God, the centre of the universe [13].

The use of IGP throughout the Islamic world expressed a strong intuition of a universe based upon logic and order. The generating rules of this art were found on a strong mathematical background. The work of Euclid and other Greek mathematicians were well known in the Islamic world [13] [42] [17]. This decorative art is generated from a discrete geometrical unit using the circle as its basis and then applying the principles of repetition, and symmetry to it [13] [17]. The basic grids are constructed based on a systematic formation of circles by connecting the circles' intersection points [111]. It is possible to generate any regular polygon by equally dividing the circumference to the required number of parts [17]. Although each pattern has its distinct geometrical design, the vast varieties of ornamental compositions are based on a simple constitutive geometry, which is generated from a limited number of simple base grids of polygons [96]. All Islamic designs were constructed by only using a compass and a straightedge; therefore, the circle becomes the foundation for Islamic patterns [13], [42], [17]. The repeat unit is constructed using a single polygon. The star unit is formed by an array of lines connecting either the intersection points of the polygon's sides or connecting the midpoints of the polygon's sides. The repeat star can be as simple as one array of lines or a combination of two or more simple stars (arrays) [50]. Combining the repeated units with the basic grid will generate the final pattern. The basic grid will be filled in with any choice of repeat star unit, and then the lines of all-stars will be extended to meet other lines at the symmetrical lines. The result will not show the basic grid, but only the replication of the star units. Historically, these patterns were never merely drawn as lines. Often, the lines are thickened when incorporated into different materials and sometimes broken up to suggest an interlacing pattern [96]. Islamic geometric patterns were applied to all kinds of materials: metalwork, woodworks, ceramics, textiles, carpets, stone, fabric, and miniatures. The universal application of these patterns implies that they were created based on solid formal methods [17].

Applications

Most of the IGP studied appear in the books by Dourgoin [9], Critchlow [13], El-Said & Farman [63], and Wade [73]. Islamic patterns may, of course, be enjoyed purely as decorations. They can be enjoyed for the aesthetic experience of art and science in unity. But these patterns depict a variety of geometrical structures and constraints of the Euclidean space. Hence, they are of great merit as educational aids for the teaching of many topics in mathematics, physics, chemistry, crystallography, computer science, and design. They can be valuable in the teaching of geometry to school children and provide a visual gateway for the teaching of abstract notions of group theory at the university level [45].

Niman and Norman [83] presented IGP as scientific and expressed that the use of two-dimensional Euclidean geometry in artistic modes of expression was most pronounced in IGP. It constituted the basis for the foundation of the rhythmic and intertwining pattern. This feature of IGP makes it a valuable means of teaching mathematical topics such as tessellation, algebra, and symmetry which leaves many opportunities for analysis and design. They elaborated that the subject matter becomes alive when presented through this medium. The other man-made two-dimensional periodical patterns suitable for the study of plane groups are primarily the Roman, Byzantine, and Romanesque mosaics, and gothic as well as some later vaulting plans and patterns.

It was not until the 1980s that mathematical research geared some of its interest in exploring the field of IGP as a method for the study of symmetry and mathematics. They noted that the most exciting developments related to this field were only twenty years old. In 1987 they wrote a book that explores geometric patterns. It deals with the mathematical analysis of tiling using regular polygons [2]. Since 1990 much research has been done to study and analyse the evolution of the Islamic symmetric repeat patterns, and to explore ways in which these patterns can be constructed algorithmically. Conducted an analytical study of different group patterns, through which they developed an algorithm based on a group theoretic approach to be used with computer graphics to generate two-dimensional symmetric periodic patterns [84]. Abas and Salman's analysis to develop mathematical tools for two-dimensional pattern analyses using planar symmetry groups and Cayley diagram [94]. He discussed the relationship between the geometry of reflection in a line and specular reflection in a mirror [98]. Developed theories discussing the formation and analysis of mosaics, lattices, and 2D plain ornaments, and explored the relationships between tessellations and the different manifolds [99] [93].

Recently, these patterns have evoked a new "humanistic" approach to mathematics education. Humanistic mathematics is a philosophy of teaching mathematics, which guides students through mathematical ideas using imagery, history, as well as other interdisciplinary Media [45], [112]. Islamic geometric patterns provide a vast ready-made stock of material for the teaching of symmetry at all levels – from kindergarten to university. Children, from a very early age, love colouring IGP may be utilized to introduce symmetry at the tender age of 3 or 4. At the university level, the IGP may be used to teach transformation geometry. The mathematical process involved in the creation of these patterns involves the application of symmetry transformations (rotations, reflections, translations, and glide reflections) on a two-dimensional template motif. These transformations form a group and hence these patterns provide a visual gateway to learning about the abstract ideas of group theory which is the key tool in the mathematical description of the interactions of the fundamental forces of nature [45]. Niman and Norman [83] presented that the feature of IGP makes it a valuable means of teaching mathematical topics such as tessellation, algebra, and symmetry which leaves many opportunities for analysis and design. They elaborated that the subject matter becomes alive when presented through this medium. The other man-made two-dimensional periodical patterns suitable for the study of plane groups are primarily the Roman, Byzantine, and Romanesque mosaics, and gothic as well as some later vaulting plans and patterns.

In the field of computer graphics, many tools and algorithms were developed for automatic construction, drawing, and visualizing of patterns. Recently research in computer graphics adapted the technique of reconstructing geometric patterns by placing star units within a basic grid of polygons. At the second SIGGRAPH conference presented a system for drawing figures constrained to seventeen ornamental design types [109]. The development of mathematical software "Tess" which can be used to interactively generate different planar ornamental tessellations by applying the rules of symmetry and the mathematical software Kali [113]. Arabeske software is a Java tool designed to help construct and draw patterns. It is particularly aimed at 3D artists wishing to use complex patterns for their scene settings. It can also be used as a special 2D drawing tool to design original patterns. It relies on the heavy use of symmetry groups [114]. He developed many related computer algorithms. In 1996 he examined the synthesis of frieze patterns and the application of these symmetry patterns in computer graphics [110]. He presented a method for constructing transformable IGP by using different star unit within the same basic grid of polygons. His construction was based on Haskin's method, which uses a grid of polygons as the basic grid [101]. He showed how to create a variety of patterns using hierarchies of symmetry and image-processing actions [100].

Computer Graphics

The IGP and symmetries were studied using the CAD package archived to make it possible to recreate these patterns accurately. In the last twenty years, many advances have been made in the field of computer vision and computer graphics [117]. The interests and tendencies of motivations in generated computer graphics between researchers have increased in recent years. There are various methods for computer graphics in some applications such as architectural arts. Computer graphics is concerned with the graphical representation of conceptual ideas or mathematical descriptions. It deals with building images from non-iconic information [102]. The advances in this field have made it possible to generate accurate and large-scale pictorial databases [Vasaros and Divinyi04]. Visualization refers to the creation of visual models of an object or process to examine it further [115]. Mechanical devices are used for translating the perceptual inputs into a geometric model interpreting colour contrast [116]. This study explores one possible solution, through utilizing digital techniques for reconstructing detailed historical IGP. Its main hypothesis is that digital techniques offer many advantages over the human eye in terms of recognizing subtle differences in light and colour [91].

The use of modern technology such as graphic software in identifying the motif of design and generating images using CAD techniques makes it easy to design images. It can be convenient to create a motif, or a complete decoration, using the capability of a conventional CAD system [108]. In other studies, present a methodology for graphic pattern design and redesign applicable to tile and textile patterns. This methodology is used in a design information system whose reference framework is the scientific theory of symmetry groups. It is reported that this information system has two computer tools:

- one for the structural analysis of graphic designs
- and another interactive tool for the structural edition of patterns that, using the presented methodology, exploit all the capabilities provided by the manipulation of the minimum region (MR) of pattern designs.

It presents some application examples to generate new designs as modifications from designs acquired from historic sources [117].

Kaplan explored the power to further the study and practice of ornament stems from three sources. He investigated the new mathematical tools: a modern conception of geometry that enables researchers to describe with precision what designers of the past could only hint at. He also investigated the use of new algorithmic tools: computers and the abstract mathematical processing they enable researchers to perform calculations that were intractable in previous generations. Finally, he reported the use of technological tools: manufacturing devices that can turn a synthetic description provided by a computer into a real-world artifact. In his article presented new opportunities for the application of computers to the analysis and creation of ornament. He presented his research in computer-generated geometric art and ornament [31]. Each of these venues represents a new research opportunity ready to be explored. In general, the field of digital imaging includes two main applications: the use of computer graphics tools to generate images such as CAD, visualization, and animation systems, etc., and the use of computers to operate on acquired images such as image analysis and computer vision [106].

Computer graphic techniques are employed to solve problems by building virtual reconstructions [117]. Successful techniques usually are very specific and directed to solving a very specialized interest. Nevertheless, existing technologies can also be adapted to solve research problems [106]. The increasing reliance on such fields is due to the development of many new techniques for data acquisition, data processing, computer vision, and computer graphics [117]. This is due to the price decline in digital computing technologies and advancements in capabilities [107]. Graphics-related research primarily focuses on the mathematical foundations of computer graphics. Such research explores and develops new approaches to modelling, rendering, simulation, and scientific visualization, and is highly connected to our work on human/computer interaction. New methods are needed to increase modelling fidelity, “fluency”, and interactivity. This is accomplished using mathematical principles from differential geometry, constrained optimization, integral equations, and piecewise differential equations, as well as physical principles such as the mechanics of solids and the physics of light.

Consistency Analysis

Consistency analysis is concerned with the problem of interpreting the entire image from local measurements. One approach to the solution is using consistent labelling. This technique uses labelling to mark features that are consistently based on some defined. Another approach to the solution is using parametric transformation, which assumes that the object we are looking for can be described by a mathematical expression classification and matching are the process of making decisions about the measurement [105]. Consistency analysis is used extensively for inferring structured features. The structural approach to feature extraction assumes that pattern structure is composed of spatial arrangements of repeated units. In this case, structure extraction becomes the task of identifying feature locations and quantifying their spatial arrangements [102]. A recognition system must contain some knowledge about the object it is trying to recognize. This knowledge might be programmed in terms of feature descriptions, or a library of shapes [104]. The output of the classification process is a decision about the class to which the recognized objects belong [102]. The extracted objects are assigned to different groups based on their vector measurements [102] and [103].

Computational Geometry

Computational geometry is a branch of computer science devoted to the study of algorithms, which stated in terms of geometry. Some purely geometrical problems arise out of the study of computational geometric algorithms, and such problems are considered part of computational geometry. Computational complexity is central to computational geometry, with great practical significance if algorithms are used on very large datasets containing tens or hundreds of millions of points Computational geometry is also concerned with the design and analysis of algorithms for geometrical patterns. Also, other more practically oriented areas of computer science— such as computer graphics, computer-aided design, robotics, pattern recognition, and operations research give rise to problems that inherently are geometrical. This is one reason computational geometry has attracted enormous research interest in the past decade and is a well-established area today.

Conclusion

This literature review was conducted by studying the various papers on IGP published in seminars, journals, and other technical publications. The papers selected are from the most relevant publications by considering the authenticity and authority of the original works and the writers. This research is trying to classify the available relevant works on the subject into different headings, e.g., the history of Islamic art, revealing the origins and evolution of the Islamic art, the symbolism meant by the original developers, sacred meanings related to theology, the consequent controversies regarding the sacred meanings, development of the theory of symmetry & asymmetry, psychology behind the symmetry, development of the group theory and its diversifications, various methods of designs and characteristics, different methods of design constructions and applications, development and growth of computer graphics, classification labelling and related consistency analysis, and finally the latest computational geometry which discussed in the light of the referred research papers. The collection and review of the referred literature will help aspiring researchers in this field.

Chapter 2: Background

In recent years we have seen a revived interest in geometric pattern designs of what was once a neglected area of research [5]. The IGP designs like art are hard to define but we may identify some of its more common features based on the definitions that have been offered in the past. The new-found interest is concerned with the mathematics of GP designs [45], the spiritual dimension of their symbolism [20], and the aesthetic aspects of their spatial and tonal domains [16]. Critchlow was among the first to analyse a geometric pattern design from a metaphysical and cosmological point of view [13]. Islamic geometric pattern designs have been viewed as an oriental art inspired by the classical Greek ruler/compass method. Rozsa Erzsebet expressed that within a relatively short period-about 100 years the Muslims conquered the Arabic peninsula, North Africa, Spain, Southern France, Sicily, and penetrated central Asia and India. Several significant cultures were flourishing in these areas at the time, and their effects on the Muslim world and thought gave birth to such works of art [47].

The scholarly study and criticism of a IGP designs are somewhat more recent but still go back at least to the ancient Roman architect Marcus Vitruvius in criticizing art decoration. Owen Jones's proposition shows many comments on the structure and features of decoration, but no definition [42]. Racinet teaches by example rather than by principle [123]. Keith Critchlow [13], J. Bourgoin [119], Issam El-Said [17], and others teach very effectively by principles. Bourgoin published his analysis of GP designs in France in 1879. The detailed works of Frithjof Schuon have been a source of inspiration and guidance for many researchers [6]. This research will provide a new framework to formulate an algorithmic language for a geometric pattern classification and design. It will use the established techniques of Martin Lings on transformation geometry [22], T. Pavlidis on algorithms for graphics and image processing [11], and Doris Schattschneider on the plane symmetry groups and their recognition and notation [69].

Sacred Meanings

Hossein Nasr explained the importance of accomplishing the internal balance of Islamic art: Islamic art is predominantly a balance between pure geometric form and what can be called fundamental biomorphic form: a polarization that has associative values with the four philosophical and experiential qualities of cold and dry representing the crystallization in geometric form- and hot and moist – representing the formative forces behind the vegetative and vascular form. [13]. One very important aspect and highly controversial issue in IGP are the sacred meanings. There is some controversy in the question of why IGP tend so strongly towards the geometric concept. Many scholars assert that this tendency is due to a strict Muslim prohibition on representation in art. One claim made is that representation is the sole dominion of Allah, who by one of His many names is known as the Giver of Form (*Al-Mussawir*). This dogmatic position can easily be refuted by the traditional Islamic arts of portraiture and miniature paintings. The only appropriate means of religious exaltation lies in art with mathematical, crystalline perfection [70]. Interpretation of IGP sacred meanings by eminent scholars has been marked by the constant pull of opposing forces.

According to Chorbachi an active group of international mystics supported specific publications and pushed certain mystical ideas in the period when oil money flowed, and art historians attempted to lure this money to their field. He also expressed that if the authors of these books, that are now readily available in the market had made clear that the presented views are modern understanding of old forms, turning them into symbols, there would be no reason to object. Furthermore, he intricates that the problem lies in presenting these modern mystical views as historical truth as if these symbols were the meanings at the time the art forms were created [56]. Gulru Necipoglu [51], Oleg Grabar [46], Doris Behrens-Abouseif [63] consider IGP as purely geometric phenomena and a typical design form of artistic expressions. Another consistency can be traced in the architectural writings, the treatise on architecture (*Risale-i-Mi'mariyye*), opens with a cosmological recount of the creation and structure of the world. Yet the text does not provide theoretical articulation of the relationship between cosmology and architecture, nor indeed do other premodern Islamic sources. The discursive relationship was mainly the work of modern theorists working with the notion of symbolism.

Another school of thought believes IGP designs cannot be viewed purely as a tangible or physical art form. Behind every IGP there is a deep and subtle underlying sacred meaning that must be properly understood against the backdrop of the holy revelations of the Quran. The foreword written by Nasr in Critchlow's book says "*it is only during the past few years that at last a few scholars in the west are becoming aware of the fact that Islamic geometric patterns are one of the most powerful forms of sacred arts and not just abstract art, in the modern sense of the word*" [13]. Seyyed Hossein Nasr is calling to educate modern man to understand the language of symbolism to revitalize traditional sciences, as he stated that such a revitalization of the traditional sciences, requires a re-discovery of the true meaning of symbolism and the education of modern man to understand the language of symbolism in the same way that he is taught to master the languages of logic or mathematics [5]. Titus Burckhardt [7], Martin Lings [22], Nader Ardalan [23], Laleh Bakhtiar [120], Adrian Snodgrass [24], Annemarie Schimmel [25], Henry Corbin [122], Louis Massignon [27], Toshihiko Izutsu [28], Hellmut Ritter [29], Mircea Eliade and Philip Mairet [30], Rene Guenon [121], Ananda Coomaraswamy [21], Frithjof Schuon [6] and others have concluded that there are cosmic symbolisms and sacred meanings that referred to IGP. This controversy has encouraged the researcher to validate the existence of sacred meaning for IGP by researching into the backdrop of Islamic culture and ancient books to establish another dimension in the language of symbolism.

Sacred is a word related to the divine or the activity related to divine rituals or places of worship. Sacred is defined as belonging to or dedicated to God; holy. Also connected with religion, sacred music, sacred writing, prayer is a sacred duty. What is called sacred may be an object? The objects chosen to be sacred are those, which are related to unknown forces [55]. Islamic spirituality could not but develop a sacred art in conformity with its own revealed form as well as with its essence. The doctrine of unity, which is central to the Islamic revelation, combined with the nomadic spirituality, which Islam made its own brought into being an iconic art wherein the spiritual world was reflected in the sensible world, not through various iconic forms but geometry and rhythm, through arabesque and calligraphy, which directly reflect the worlds above and ultimately the supernal sun of divine unity [13]. The work of the artists expresses the inspiration of the experts to the design of Islamic art and architecture and the expressions of a faith that forbids direct depictions of holy figures [53].

The use of IGP is one of the chief characteristics that give the Islamic artistic heritage its distinct identity. Geometry as an abstract art form was developed in part due to the discouragement of images in Islam on the basis that it could lead to idolatry [50]. Cosmic creations of the Almighty are the most fundamental inspirations behind the evolution of geometrical shapes in IGP language. Forms and shapes of cosmic and celestial bodies have inspired the designers of the IGP and are the primary icons of expression of the creative urge beneath their thoughts. Expressing these concepts was never a subjective matter. Islamic artists did not seek to express themselves as such but rather aimed to honor matter and reveal the objective nature of its meaning [13], [57].

The two ancient texts The Description of Benefits (Sherh Al-Fewaaed in-Arabic) [58] and The Great Knowledge of Sun (Shems Al-Meaaref Al-Kubra in-Arabic) [59] are the works of neither by any one of the current age nor of any university department. Ancient experts had written them several hundreds of years ago. Both these books are important watersheds in deciphering the sacred significance behind Islamic geometric designs. They describe the relationship and potential metaphysical power between IGP and sacred numerals, geometric shapes, cosmic objects, and alphabets. Also, clarify the thought process of the artisans of those ages. The relevant texts of both these books have been translated into English from Arabic by the researcher.

Muslims made remarkable achievements in the fields of astronomy, mathematics, and geometry. In Islam, the main sources of cosmological ideas were naturally the Quran and the prophetic sayings (*Hadith*). The new findings contributed to their cosmological thinking. Persian, Indian, and other Near Eastern influences were absorbed into the Muslim worldview; however, it was Greek knowledge that ultimately prevailed in the Islamic world. Islamic mysticism reached its zenith in the work of Ibn-Arabi (d. 1240), whose cosmology has since dominated the Islamic world. Cosmology from a pre-modern Sufi perspective assumes some knowledge in the Islamic cosmological thinking, of the sense in which cosmology can be related to GP designs in Islamic art and of mystical thoughts in which the relationship is grounded.

Geometrical shapes derive their attributes from cosmic entities. Shapes and dimensions have fascinated man since time immemorial. The first important geometric shape in IGP grid system is the circle. The sun among the celestial bodies visible to the astronomers of antiquity became symbolic of the supreme authority of the Creator Himself. To the Egyptians, the sun was the symbol of immortality, for, while it died each night, it rose again with each ensuing dawn. The celestial spheres, or celestial orbs, were the fundamental entities of the cosmological models developed by Plato, Eudoxus, Aristotle, Ptolemy, Copernicus, and others. The geocentric model created by Greek astronomers assumed that the celestial bodies moving about the Earth followed perfectly circular paths. The circle was regarded by Islamic mathematicians and philosophers as the perfect geometric figure and consequently the only one appropriate for a geometric pattern design as shown in figure 1.

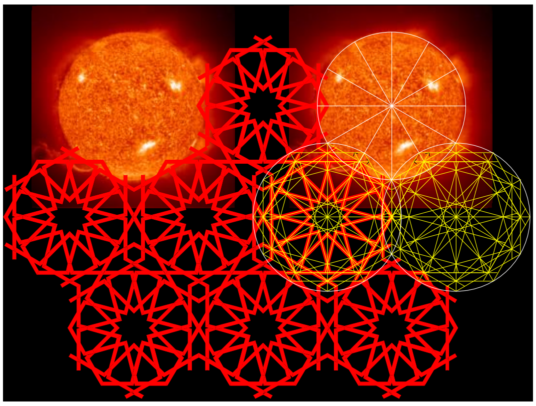


Figure 1: The sun symbolic of the supreme authority of the Creator Himself

The second important geometric shape in IGP grid system is the square. Prophet Abraham built a perfect cubical Kaaba; the most popular symbol associated with Islam. The cubical Kaaba was built thousands of years before 622 A.D. the beginning of Islam, as it is stated by the Holy-Quran:

"*And [mention] when Abraham was raising the foundations of the House and [with him] Ishmael, [saying], "Our Lord, accept [this] from us. Indeed, You are the Hearing, the Knowing*" [60, 2:127].

وَإِذْ يَرْفَعُ إِبْرَٰهِـۧمُ ٱلْقَوَاعِدَ مِنَ ٱلْبَيْتِ وَإِسْمَـٰعِيلُ رَبَّنَا تَقَبَّلْ مِنَّآ ۖ إِنَّكَ أَنتَ ٱلسَّمِيعُ ٱلْعَلِيمُ ﴿١٢٧﴾

God has commanded the pilgrims to replicate the circular motion of celestial bodies on earth; the circular motions of faithful around the cubical Kaaba as shown in figure 2.

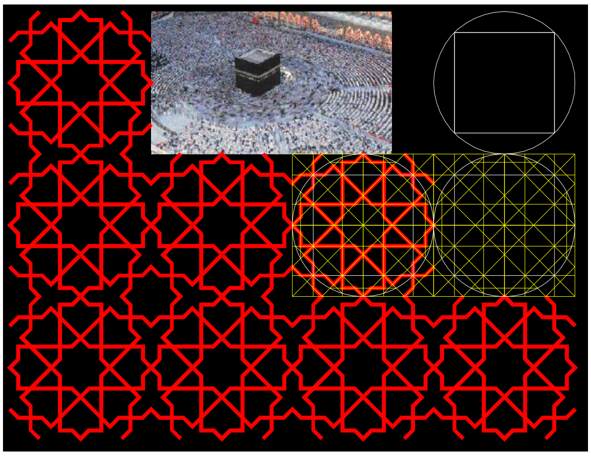


Figure 2: The circular motions of faithful around the cubical Kaaba

The third important geometric shape in IGP grid system is the triangle. The triangle represents the numeral three, and when three gives rise to six it makes a six-pointed geometric star, in which two triangles are mounted on top of each other in opposite directions as shown in figure 3. The Holy-Quran states the creation of heavens and earth in six days symbolizing the perfection.

"*And We did certainly create the heavens and earth and what is between them in six days, and there touched Us no weariness*" [60, Qaf 38].

وَلَقَدْ خَلَقْنَا ٱلسَّمَـٰوَٰتِ وَٱلْأَرْضَ وَمَا بَيْنَهُمَا فِى سِتَّةِ أَيَّامٍ وَمَا مَسَّنَا مِن لُّغُوبٍ ﴿٣٨﴾

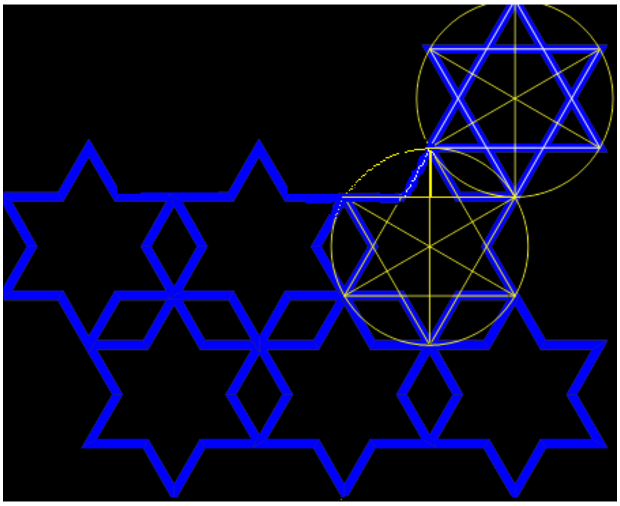


Figure 3: The creation of heavens and earth in six days symbolizing perfection

As illustrated, various grid systems that are used in IGP designs are inspired from different celestial bodies. Certain rituals like the motion of the pilgrims in a circular fashion around the cubical Kaaba can be associated with IGP grid system and it was imperative for the designers to use them creatively and is the primary icons of expression of the creative urge beneath their thoughts. Also, the triangle and hexagon are strongly associated with the theological symbols of the people of the book (Abrahamic religions). To conclude, until the accomplishment of modern physics, cosmology was the choice of theologians, mystics, and philosophers, forming the core of religious sciences. Celestial bodies are the most fundamental inspirations behind the evolution of designs. We have seen that the usage of sacred icons in religious art is far and wide across major religions of the world. We have also observed that icons hold a lot of significance through their symbolic power and that there indeed are sacred meanings to icons whenever they are in the context of any religion. Islam has a historic relationship with icons and has effectively utilized them in art.

Sufism

Sufism is an Islamic phenomenon associated with spirituality that emerged in the early formative period. Some scholars trace it back to the Prophet Mohammed and his immediate companions. Ibn-Khaldun (d.1406), the celebrated Andalusian historian, regards Sufism and Islam as two equal terms. The piety and devotion of the first generation of Muslims provided, in his view, the model for Sufism. Other Sufi masters, such as Ibn-Arabi, Rumi, and al-Jili produced a wealth of religious and mystical literature and profound poetry that were to shape the intellectual life of Muslim communities until the dawn of the twentieth century. Since its emergence Sufism has continued to play a significant role in the intellectual, socio-cultural, and political life of Muslim communities. Alexander D. Knysh observes that the study of the artistic life in premodern Islam without considering Sufism is equally distorting [148]. In recent decades Sufism has gained noticeable popularity in the West and this been the subject of growing scholarly interest, however, many key Sufi texts have remained unavailable, let alone good translations.

Al-Boni Sacred Meanings

Shems Al-Meaaref Al-Kubra (*The Great Knowledge of Sun*) is an ancient sacred text by Ahmed Bin Ali Al-Boni reveals that there is hidden meaning to all numerals and geometric shapes, wherein he wrote the association of angels to certain numerical attributes of various geometrical shapes, four angels are glorifying God [59, p. 43]:

1. Heptagon for the angel Gabriel (is the conveyer of the message to Prophets).
2. Square for the angel Israfiel (is the owner of forms and blows. He has three blows; the blow of fear, the blow of death, and blow of revival. Each blow has its affairs).
3. Triangle for the angel Azraiel (is entrusted with taking souls and their vanishing).
4. Octagon for the angel Michaiel (is entrusted with the means of subsistence for worshippers).

He points out the subtle significance of numbers and alphabets in the context of religion and associates the celestial planets with geometrical shapes [59, p. 9]:

1. Pentagon matches Saturn,
2. Hexagon matches Jupiter,
3. Heptagon matches Venus,
4. Octagon matches Mercury and
5. Nonagon matches Moon

He designates the twelve signs of the zodiac in heaven and their four materialistic natures as there are twelve zodiacs in the heaven: Aries, Leo, Sagittarius, Gemini, Libra, Aquarius, Scorpion, Cancer, Pisces, Capricorn, Taurus, and Virgo, which are divided to four natures, fire, air, water, and soil [59, p. 336] as shown in figure 4.

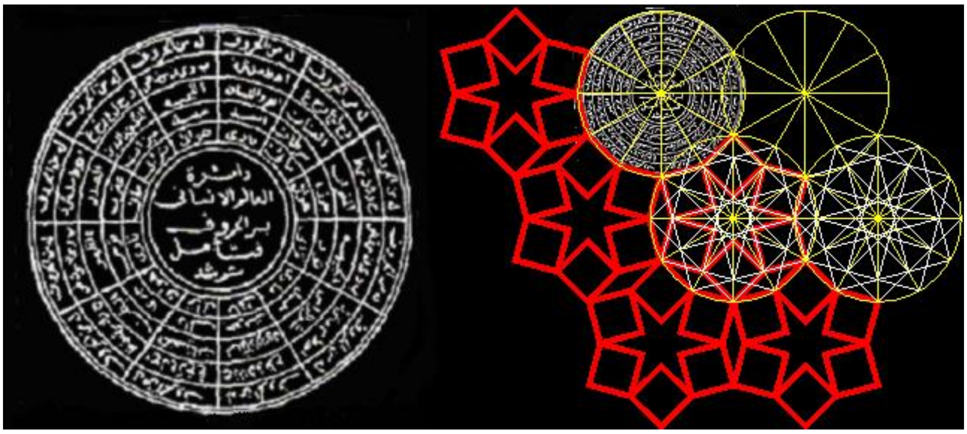


Figure 4: The twelve signs of the zodiac in heaven

According to Al-Boni the sacred square and its divisions have the power to change the material destiny as given in the following paragraph: It is said that in the middle of Solomon's carpet there were four Hebrew names of Jinn's (hesh-tesh-leh-koosh, kesh-kesh-lioosh, bekh-leh-lesh-toosh, and shet-let-tesh-koosh). These names have a great effect and are highly beneficial for everything you wish [59, p. 55] as shown in figure 5.

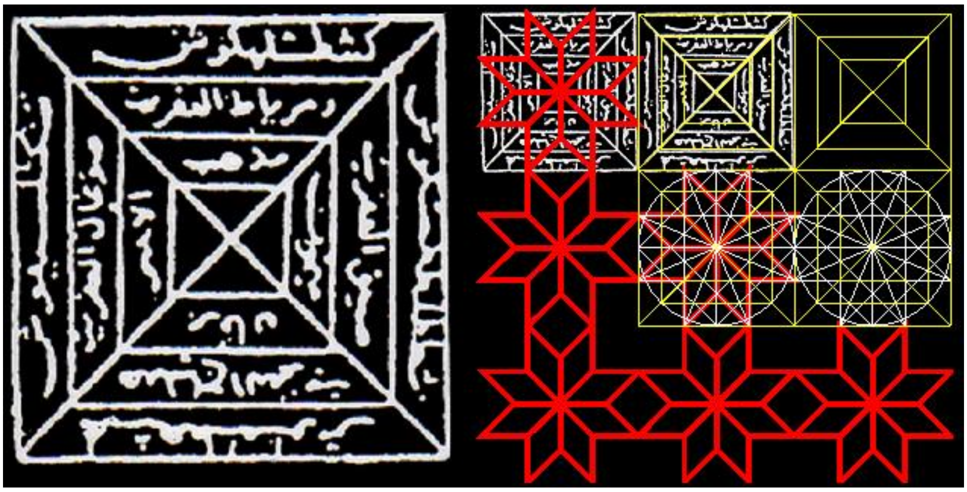


Figure 5: In the middle of Solomon's carpet there were four Hebrew names of Jinn's

Al-Boni reveals graphically with the design of the circle and the square the possibility of a miracle in the physical world; Whoever writes the letter "meem" "m", its shape and form, and carries it with him and went to the kings, rulers, judges, he will be accepted by them and influential [59, p. 336] as shown in figure 6.

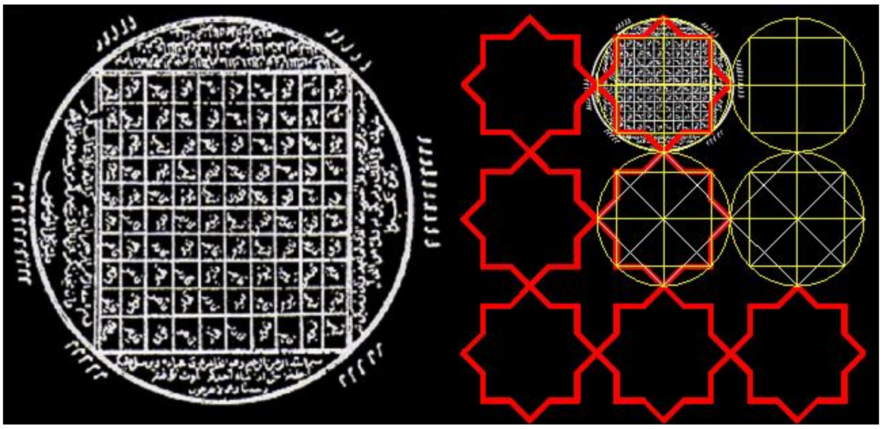


Figure 6: The letter "meem" "m" with circle and square have miracle in the physical world

Al-Boni reveals the preserved circle is the circle of existence which includes the secrets of existing things. It was reported about Imam Ali who reported about the Prophet Mohammed, who told him this secret about this shape. This circle contains the names of kings, rulers, states, and wars. This is among the convention of breaking and spreading the letters, and striking every origin by its roots, as each character of his name is associated with the proper number and when its class and form along with the country of origin is checked, you will know what the future happenings of that country would be [59, p. 348] as shown in figure 7.

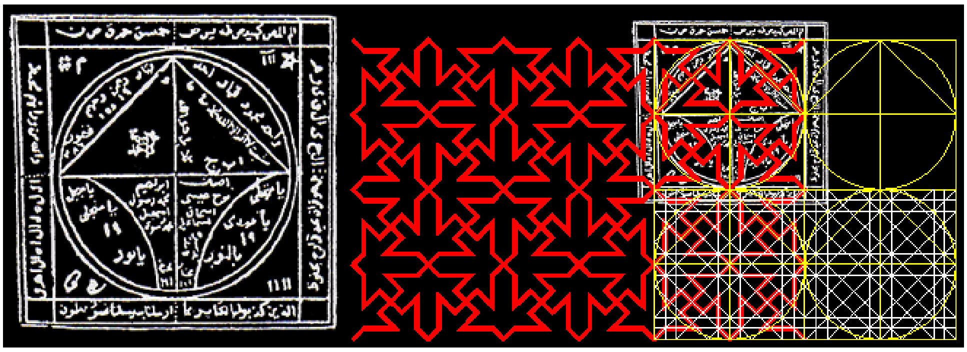


Figure 7: The circle of existence which includes the secrets of existing things

Al-Ahsai Sacred Meanings

Shems Al-Meaaref Al-Kubra (*The Great Knowledge of Sun*) is an ancient sacred text by Ahmed Bin Ali Al-Boni there is hidden meaning to all numerals and geometric shapes, wherein he wrote the association of

Sherh-al-Fewaaid (*The Description of Benefits*) is another ancient Sufi sacred text by Ahmad Bin Zain-Adeen Al-Ahsai (1753-1826) explains that "*one may think that these matters are occasions on which the secrets of the creation are not based. However, they are secrets hidden by Almighty God with a shelter of unknown things and manifested their effects in His creation and made the effects prove the secrets*". Al-Ahsai articulated the association of sacred numerals 1 to 9 in terms of the existence of sacred worlds from the doom of creation [58] wherein he wrote:

1. **One World (The Centre – The Point)**

   Presents God; (Alem-al-Wahed)

   One World and not the World-of-God, because God cannot be in a Thing-World, which would mean that the 'Thing' that surrounds God has more power. [58, p. 90].

2. **Two Worlds (The Line in Geometry)**

2.1 World of Unseen (Alem-al-Qaib)
2.2 World of Seen (Alem-al-Shehadeh)

The Islamic philosophy consisted of The Seen and Unseen, the Divine, and Human Domains. This basic understanding assumed in the Western tradition a decisive philosophical formality in the spiritual distinction between the Sensible and the Intelligible. Medieval Muslim scholars, who inherited and developed the intellectual tools of Greek philosophy, maintained this through the divide between (Al-Hissi) and (Al-Aqli). This was, in a sense, legitimized by the Quranic polarity of the (Seen) and the (*Unseen*) [58, p. 90].

3. **Three Worlds (The Triangle in Geometry)**

3.1 World of Necessity (Alem-al-Wojoob) it is the world of eternal almighty.
3.2 World of Superiority (Alem-al-Rejehan) it is the world of desire, will, and creativity.
3.3 World of Legality (Alem-al-Jewaz) it is the world of restrictive existence, its beginning is the mind (Al-Dura), and its end is molecules or earth (Al-Thera) [58, p. 90-91].

4. **Four Worlds (The Square in Geometry)**

4.1 World of Creation (Alem-al-Khelq)
4.2 World of Sustenance (Alem-al-Rezq)
4.3 World of Death (Alem-al-Mout)
4.4 World of Life (Alem-al-Heyat) [58, p. 91-92].

5. **Five Worlds (The Pentagon in Geometry)**

5.1 World of Eternity (Alem-al-Azael)
This is the world of indicating eternal almighty God.

5.2 World of Immortality (Alem-al-Sermed)
This is the world of order and will.

5.3 World of Mightiness (Alem-al-Jeberout)
This is the world of meanings free of matter, form, and period.

5.4 World of Realm (Alem-al-Melekout)
This is the world of form free of matter and period.

5.5 World of Dominion (Alem-al-Moolk)
This is the world of indicating its beginning of a specific direction and its end is the earth [58, p. 92].

6. **Six Worlds (The Hexagon in Geometry)**

6.1 World of Minds (Alem-al-Oqool)
This is the world of essential meanings, selves-free of element substance, psychological form, time-period, and embodiment.

6.2 World of Psyches (Alem Al-Nofoos)
This is the world of essential structures, which are the words of the preserved ledger and the written Book.

6.3 World of Natures (Alem-al-Tebiaa)
This is the world of dissolution and breaking after formation, i.e., things after completing their discrimination. They are broken and dissolved until their upper equals their lower, their visible aspects equal their hidden aspects, their strong equals the weak, their humid equals the dry and the hot equals the cold.

6.4 World of Dust (Alem-al-Hibaa)
This is the world of molecules in the air from (al-Toor), mountain in Sinai, which is turned by God into, levelled ground, and this is the molecular existence of every particle in every creation of God.

6.5 World of Embodiment (Alem-al-Mithal)

This is the world of existing forms in the interval air (Alem-Hewaa-al-Berzekh), which means between the realm and the dominion, bodies without souls with their face to age and their back to time.

6.6 World of Bodies (Alem-al-Jism)

This is the world formed of elemental matters and ideal forms.

In another example says: “Heavens and earth evolved in different stages like sperm (Nutfah), clot (Aleqeh), lump-embryo (Mothqeh), bones (Atham), bones covered with flesh (Lehm) and then another creature created (Khelq) [58, p. 94-95].

7. **Seven Worlds (The Heptagon in Geometry)**

7.1 World of Fire (Alem-al-Nar)
7.2 World of Air (Alem-al-Hewaa)
7.3 World of Water (Alem-al-Maa)
7.4 World of Soil (Alem-al-Turab)
7.5 World of Body (Alem-al-Jism)
7.6 World of Psyche (Alem-al-Nefs)
7.7 World of Soul (Alem-al-Rooh)

The seven worlds are presented as every event has a triangular existence (Mutheleth Al-Keyan) and a quadrilateral nature (Murebaa Al-Keifiya); meaning everything that is created has three existences (Body, Psyche, Soul) and four natures (Heat, Humidity, Coldness, Solidness) [58, p. 92,96].

8. **Eight Worlds (The Octagon in Geometry)**

8.1 World of Creation (Alem-al-Khelq)
8.2 World of Sustenance (Alem-al-Rezq)
8.3 World of Death (Alem-al-Mout)
8.4 World of Life (Alem-al-Heyat)

If considered in the present and afterlife would constitute (Eight Worlds). This means the joining of the Four Worlds in the present life and the four worlds in the afterlife as carriers of the Throne (al-Arsh) of God on resurrection day [58, p. 98-99].

9. **Nine Worlds (The Nonagon in Geometry)**

9.1 World of Partial Hearts (Alem-al-Qoloob-al-Jozaeya)

this is the world of the element of the total heart.

9.2 World of Partial Psyches (Alem-al-Nofoos-al-Jozaeya)

This is the world of the land of heavenly people which is from the firm orbit.

9.3 World of Partial Minds (Alem-al-Oqool-al-Jozaeya)

This is the world of wisdom from Saturn’s orbit.

9.4 World of Sciences (Alem-al-Oloom)

This is the world of forms of information as they are.

9.5 World of Illusions (Alem-al-Awham)

This is the world of which is the principle of psychological structures, which is from Mars’s orbit.

9.6 World of Second Existences (Alem-al-Wojodat-al-Thaniya)

This is the world of corporal existence made of matter and form from the Sun’s orbit).

9.7 World of Imagination (Alem-al-Kheyalat)

This is the world from Venus’ orbit).

9.8 World of Thoughts (Alem-al-Afkar)

This is the world from Mercury’s orbit.

9.9 World of Sensational Animal Life (Alem-al-Heyyat-al-Heiwaneya-al-Hesiyah)
This is the world of sensational animal life, which is common between all animals and is from Moon's orbit [58, p. 98-99].

Critchlow Sacred Meanings

The foreword is written by Nasr in Critchlow's book Islamic Patterns: An Analytical and Cosmological Approach to comprehend the sacred meanings and cosmic symbolism behind IGP suggest that … "*It is only during the past few years that at last a few scholars in the West are becoming aware of the fact that Islamic art is one of the most powerful forms of sacred art and not just abstract art in the modern sense of the word. Thanks to the efforts of a small number of authorities, foremost among them F. Schuon and T. Burckhardt*". He also mentions the sacred meanings of numbers and figures … "*There is certainly within the spiritual universe of Islam a dimension which is called Abrahamic Pythagoreanism, or a way of seeing numbers and figures as keys to the structure of the cosmos and as symbols of the archetypal world and a world which is viewed as the creation of God in the sense of Abrahamic monotheism*". Islamic art is gradually coming to be understood for what it is, namely a means of relating multiplicity to unity using mathematical forms which are seen, not as mental abstractions, but as reflections of the celestial archetypes within both the cosmos and the minds and souls of men [13]. Critchlow explains the spiritual meaning of numbers and shapes in the following terms:

1. **The Point**
The point is the symbol of unity and source "*The manifestation of an action, object or thought that necessitates a point of origin or departure ... serves as a symbol of unity and source. In terms of geometry, it represents the center – the elusive controlling point of all forms*" [13, p. 9-11].

2. **The Line**
Symbol of direction, and departure from source wherein he wrote: "If the manifestation of the point is indicative of a departure from its source, then the direction is implied. The direction in space is qualitative, and hence the first departure or line path from the point is qualitative. A line, i.e., when a point has moved outside and away from its original position, symbolizes the polarity of existence" [13, p. 11].

3. **The Triangle**
The triangle is the symbol of nature, and consciousness "*The triangle represents his three-fold nature and the three-fold nature of his consciousness: the knower, the known and the knowing; the object, the subject and the conjunctive; or the relationship, which links them*" [13, p. 150, 171].

4. **The Square**
The square is the symbol of earth "*The square is the representative of the "earth", or the world of the physical elements: earth, air, fire, and water – solid, liquid, gases, and radiation state of energy*". Heaven of signs of zodiac (falak-al-buruj; in-Arabic) [13, p. 150].

5. **The Pentagon**
The pentagon is the symbol of divine presences "*These five Divine Presences should be borne in mind whenever the pentagon or five-fold symmetry is referred to in this book as it is the intrinsic underlying symbolism*" [13].

   - The prime or first Presence is the Absolute Unity of God (*Allāhu-had*).
   - The second is God as Creator, Revealer, and Savior, the Divine Qualities.
   - The third is the Throne (*Al-Arsh*); formal manifestation, identified as the world in its entirety.
   - The fourth is the Footstool (*Al-Kursi*); on which the feet of God rest (this is an animistic manifestation, Rigor, and Mercy).
   - The fifth, most distant, Presence is the Earth (*Al-Arth*) and the Human Realm (*Al-Nāsūt*).

6. **The Hexagon**
The hexagon is the symbol of perfection, and six days of creation "*The hexagon, as the number of perfections, is symbolic of "heavens"; it is the natural division of the circle as each edge of the hexagon is exactly equal to the radius of the circle which contains it and can also be taken as symbolizing the six days of creation, which represents a perfection*".

Three gives rise to six, and six has a vital role in Islamic cosmology as it does in the other two Abrahamic religions, Judaism and Christianity; and that role relates to the number of days in which God created the world. The Abrahamic wisdom insists that we regard the inner and outer meaning of this cosmogony, the symbolic and literal dimensions. The six-pointed star is a symbol of perfection in all three religions [13, p. 150].

7. **The Heptagon**
The heptagon is the symbol of planetary spheres "*Next in importance in the philosophical perspective of the world in which the divine intellect is filtered through seven spheres of the planetary world … (Sun, Saturn, Mars, Venus, Jupiter, Moon, and Mercury) … Seven not only represents the number of planetary spheres but is the quarter division of the lunar cycle of twenty-eight days, and becomes the rhythm of a week, intimately related to the work and rest cycle in all Abrahamic religions*" [13, p. 59].

8. **The Octagon**
The octagon is the symbol of signs of zodiac "*Heaven of Signs of Zodiac (falak-al-buruj; in-Arabic)*" [13, p. 60].

9. **The Nonagon**
The nonagon is the symbol of heaven of heavens "*Heaven of Heavens (falak-al-aflak) … Nine has a special relation to the heavens and their generating intellects according to Ibn-Sina (Avicenna), the great philosopher and physician, born near Bukhara in 370 A.H./980 A.D."* [13, p. 60].

The Holly-Quran is considered as a theorized mode which is concerned with making sense of the Quranic and prophetic material and was cultivated in three different intellectual spheres: theology and polemics philosophy and science, and hermeneutics and mysticism. The Quran presents many references to cosmic elements; the Throne, the Footstool, the Pen, the Tablet, heaven, and earth; to the creation and resurrection, to paradise and hell, and so on, but mostly in an abstract way without weaving a complete and coherent cosmic picture [60].

Fontana in his article reported that the third most important geometric shape in IGP is the triangle. The triangle represents the numeral three, and when three gives rise to six it makes a hexagon. He also stated in his book "*The Language of Symbols*" that the triangle pointing upward stands for the ascent to heaven, fire, the male principle. The triangle pointing downward stands for grace, descent from heaven, water, the female element [61]. Seyyed

Hossein Nasr explained the importance of the "outer" and "inner" meaning of Islamic art for accomplishing the internal balance [41], [50]. You may think that these matters are occasions on which the secrets of the creation are not based. However, they are secrets hidden by Almighty God with a shelter of unknown things and manifested their effects in His creation and made the effects prove the secrets [58].

Controversy over Sacred Meanings

It is often said that geometry was forced on Islamic art because Islam forbids the drawing of animate shapes. In the view of the researcher, this explanation suffices only at a rather superficial level and misses out deeper and more significant reasons. The first thing that needs to be said to challenge this simplistic repetition is that there is a vast body of figurative work by Muslim artists. Everyone has seen examples of Persian miniatures, but apart from these, there exist many realistic life-like pictures of humans as well as animals executed with great virtuosity and naturalism by Muslim artists. This is especially true for work produced during the Moghal period in India, but such works exist in other places as well [45].

Lawlor describes in his book that sacred geometry involves sacred universal patterns used in the design of everything in our reality, most often seen in sacred architecture and sacred art. The basic belief is that geometry and mathematical ratios, harmonics and proportion are also found in music, light, and cosmology. This value system is seen as widespread even in prehistory, a cultural universal of the human condition. It is considered foundational to building sacred structures such as temples, mosques, megaliths, monuments, and churches [61].

Chorbachi points out that to believe in mysticism and to follow in its practices and experience its positive effects is one thing, but to promote new mystical interpretations under the guise of historical truth, especially when no documented evidence is given, is quite another wherein he wrote; "*I wonder if the artisan who made this design thought of it as form, expansion, contraction and the Breath of the Compassionate God…Where does one draw the line between true historical research and the creation of and attribution of symbolic meaning to forms from the past? How can we redeem the geometric shapes, forms, and patterns from the shrouds of mystical interpretations to see the precise scientific design at their basis?*" [62].

Chorbachi presents sacred meanings of IGP as modern understanding and there flowed, historical truth to the meaning of these geometries at the time of design, wherein he wrote: "*In the mid-seventies …a very well-known, active group of international mystics supported specific publications and pushed certain mystical ideas…Perhaps it was no coincidence that this was the period when oil money flowed and art historians attempted to lure this money to their field. In those years, the money of the imperial courts of Persia played a very active role in the Islamic art scene. Exhibitions, conferences, and publications multiplied. If in these books, that are now readily available on the market, their authors had made clear that the presented views were modern understanding of old forms, turning them into symbols, there would be no reason to object. The problem lies in presenting these modern mystical views as historical truth as if these symbols were the meanings at the time the art forms were created*".

Doris argues that the norms of beauty in the Arab-Islamic culture were autonomous, pleasure-oriented, and independent of moral and religious criteria. Although her proposition is predicated on the lack of contrary evidence and loosely argued, her view contrasts that of the traditionalists who would ardently disagree that meanings in Islamic art and architecture are a matter of aesthetics and psychology instead of symbolism and epistemology [63].

Coomaraswamy affirms a degree of anonymity, which reflects the sense of universality their discourse promotes, "It is not the personal view of anyone that I shall try to explain, but that doctrine of art which is intrinsic to the traditionalist philosophy" [21].

Burckhardt argues, inevitably approaches Islamic art in the purely analytical way of all modern sciences, by dissection and reduction to historical circumstances. A form, though limited and consequently subject to time may convey something timeless and, in this respect, escapes historical conditions, not only in its genesis which partly belongs to a spiritual dimension but also in its preservation, to a certain extent at least [7].

Necipoglu main concerns centre on the following points: the uncritical application of the theological concept of divine unity (*tawhid*) to emphasize the unity of Islamic art and architecture; the lack of contextualized and historicized analyses; the dogmatic and essentialist approach to the subject; and the "*sweeping generalizations unsubstantiated by concrete data*" [51].

Speaking from a scientific standpoint Chorbachi wanted to dismiss the confusion that beset the field of IGP because of the lack of a common language. Chorbachi main concern is the extension of the mystical doctrine of the Unity of Being (*wandat al-wujud*) to the field of geometry, by proposing that all geometric articulations in Islamic art and architecture can be reduced to the division and subdivision of a circle, conceived, and presented as the symbol of divine unity. Chorbachi writes, this view "*is pushed to the point of scientific fallacy*" She seems rather puzzled with how simple geometric exercises in rotational symmetry or the interlocking eight-pointed star and cross pattern can become symbols of divine presence and absence or the Breath of the Compassionate [31].

Nasr is calling to educate modern man to understand the language of symbolism to revitalize traditional sciences, as he stated: "*such a revitalization of the traditional sciences, however, requires a re-discovery of the true meaning of symbolism and the education of modern man to understand the language of symbolism in the same way that he is taught to master the languages of logic or mathematics*" [14]. Driven by a keen interest in Sufi spirituality, they sought to show how in premodern Islam the mystical experience was inextricably linked to most aspects of life. Sufi spirituality they argued can be traced in various modes of expression, the most conspicuous of which are art and architecture.

Classification

The new study, published in the Journal Science, offers a window into a time when Islamic scholars were centuries ahead of their European counterparts and used their mathematical prowess to convey the holiness in art "*We're talking about a branch of math that most people didn't understand until the last 30 years*" [53]. Jeremy describes that as it doesn't arise in a culture that's unwilling to have intellectual freedom [117]. From the 10th to the 13th century AD, aided by mathematicians the artists and artisans in the Islamic civilization produced a large body of symmetric geometrical patterns [79]. If an artisan was running into a major problem decorating the king's palace, there would have been a mathematician sitting right there. There's no reason why they couldn't have been in contact [51]. Emil and Milota Makovicky classify drawings of Escher, Roman, Byzantine, Romanesque mosaics, and Arabic geometrical patterns as crystallographic classes of two-dimensional space groups [67]. Islamic art and engineering often had a theoretical basis [51]. The complex geometry in the art of the period trickled down from the achievements of mathematical scholars. Muslims in Baghdad had made the first translations of Euclid's geometry in the 9th Century, giving them a 200-year head start on European intellectuals and architects [53].

The mathematical aspect of designs has received scant attention and evoked the interest of researchers, mathematicians, geometrizes as to their classification. Mathematicians and some other scientists may find it convenient and useful to interpret regularity of a pattern in terms of its pattern group theory, i.e., 7- frieze or 17-wallpaper. The Single dimensional symmetric pattern is referred to as 7-frieze and the double dimensional symmetric pattern is referred to as the 17-wallpaper pattern group theories. Primarily E. Muller [82], H. S. M. Coxeter [131], N. V. Belov [132], Fejes Toth [133] and Martin Lings [22] referred to these patterns as wallpaper or more appropriately termed them as crystallographic patterns. In this way, they applied the results of algebra and other mathematical disciplines to the study of art pattern classification. Milota and Emil Makovicky classify drawings of Escher, Roman, Byzantine, Romanesque mosaics, and Arabic geometrical patterns as the crystallographic class of double dimensional space groups [67]. W. K. Chorbachi suggests that the scientific finding of pattern group theories is the way to understand the classification of designs [56]. Andrew Glassner presented the 7-frieze pattern group theory to classify art patterns [110]. In the notion of classification Syed Jan Abas and Amer Shaker Salman categories any double dimensional periodic pattern as one of 17-wallpaper pattern group theory [84]. Likewise, the arrangement theories proposed by Lee Xah based on the 17-wallpaper pattern group theory as benchmarks for art patterns are also too general [1]. The thesis work of Andreas Speiser considers the scientific theoretical finding and reveals the value of these scientific theories to the understating and classification of designs [81]. Frieze and wallpaper pattern group theories appear almost universally in books because they are perfect illustrations of those ideas. The pattern group theory approach to our subject is deeply satisfying because it is very elegant and presents a beautiful chain of thinking that makes the results seem inevitable and certain.

Modern studies have been heavily tilted towards viewing the classification on the yardsticks of pattern group theories. However, it could be argued that this is not the concept of regularity that designers had in mind as they were creating their art. Conventional approaches to classify art patterns like the 7-frieze and the 17-wallpaper pattern group theories were and are still being considered as the benchmarks for the classification of designs. The difference between the two approaches of pattern group theories will be analysed about IGP classification.

Classification is a mammoth task, and the state of the problem could span into geographical classification (classification based on the region of the origin of the pattern), or geometric classification (i.e., classification based on a grid system and geometry). Currently the 7-frieze and 17-wallpaper pattern group theories are considered to classify designs in terms of arrangement formats of the IGP. Emil Makovicky was of a firm opinion that symmetry can be described in exact terms of the canons or preferences exhibited in the visual art created by a given person or school. Their nature and their changes, or succession in time, appear to this author to reflect very explicitly the basic features of all human thoughts and creativity either in an individual or in entire groups/schools [134]. In the following sections, there is an attempt by the researcher to demonstrate that the current pattern group theories i.e., 7-frieze and 17-wallpaper are merely arrangement classification theories and not classifications based on the normalization grid methodology.

Frieze Group Classification

The idea of frieze pattern groups has been inspired by frieze decorative horizontal bands. The frieze pattern groups are simply abstract designs used purely for decoration. Ornamental bands are also showing in such diverse crafts such as pottery, embroidery, and furniture. When some repeating motif creates these bands, and particularly if that pattern has some internal symmetry, then there is a good chance the structure of the design can be represented mathematically as a frieze pattern groups. Designs that are invariant under all multiples of just one translation are frieze or border ornaments, and their associated groups are commonly called frieze groups [4]. Group theories shows that in a single dimension symmetric period patterns can be analysed into seven different types and provides the information needed to identify the minimal understating to generate and identify symmetry type [135].

The frieze groups are often visualized by this periodic pattern, with the group being those transformations under which the pattern is invariant. The basic idea behind frieze pattern groups is to generate different kinds of periodic patterns from a single unit pattern. Surprisingly there are only seven fundamentally different patterns that could be generated. Therefore, frieze pattern groups are only seven types. The study of frieze patterns is traditionally associated with a branch of abstract mathematics. Recognizing the four types of transformation (translation, reflection, rotation, and glide reflection) is useful to analyse the IGP pattern. A frieze group necessarily contains translations and may contain glide reflections. Other possible group elements are reflections along the long axis of the strip, reflections along the narrow axis of the strip, and 180° rotations. Two of the seven frieze groups are singly generated, four have a pair of generators, and the last of the seven requires three generators. Examples of such work can be seen in ancient Egypt story-telling frieze pieces.

Glassner used the letter (F) for Frieze in the cell because it is easy to recognize in any orientation and has no symmetries of its own. On other hand IGP has symmetry of its own. In the following sections there is an attempt by the researcher to demonstrate that the current 7-frieze pattern group theory is merely arrangement classification and not classification based on grid system as shown in figures 8-15.

1. **Frieze Group Translation**

This group of frieze symmetry consists of translation only. It is one of the basic ways to repeat the design motif infinitely to the (*left-right*) direction (or vice versa) by simply sliding some or the entire motif; the result is indistinguishable from the original band as shown in figure 8. A translation "slides" an object a fixed distance in each direction. The original object and its translation have the same shape and size, and they face the same direction; it is a direct isometry. From bellow figure we can see the effect of translation isometry on the (F) letter and IGP design stays the same.

$$T_{hk}(x, y) = (x + h, y + k)$$

Figure 8: The Effect of Frieze Translation Symmetry on IGP

2. **Frieze Horizontal Reflection**

This group of frieze symmetry consists of horizontal reflection only in an (*up-down*) direction (or vice versa) parallel with the horizon as shown in figure 9. A reflection is a flip. It is an opposite isometry; the image does not change the size, but the lettering is reversed. Reflection in the x-axis: When you reflect a point across the x-axis, the x-coordinate remains the same, but the y-coordinate is transformed into its opposite. We can see the effect of horizontal reflection isometry on the (F) letter changes but on IGP design stays the same.

$$P(x, y) \rightarrow P'(x, -y) \quad \text{or} \quad r_{x\text{-axis}}(x, y) = (x, -y)$$

Figure 9: The Effect of Frieze Horizontal Reflection Symmetry on IGP

3. **Frieze Vertical Reflection**

This group of frieze symmetry consists of vertical reflection only aligned in a (*left-right*) direction, perpendicular to the horizon as shown in figure 10. When you reflect a point across the y-axis, the y-coordinate remains the same, but the x-coordinate is transformed into its opposite. We can see the effect of vertical reflection isometry on the (F) letter changes but on IGP design stays the same.

$$P(x, y) \rightarrow P'(-x, y) \quad \text{or} \quad r_{x\text{-axis}}(x, y) = (-x, y)$$

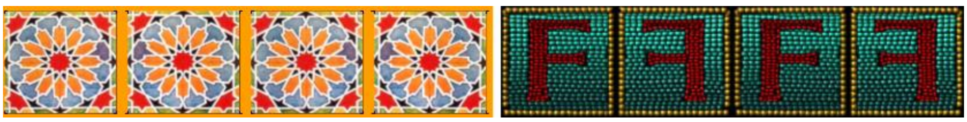


Figure 10: The Effect of Frieze Vertical Reflection Symmetry on IGP

4. **Frieze Vertical Reflection and 180° Centre Point Rotation**

This group of frieze symmetry consists of vertical reflection on the vertical line, then rotation from its centre point by 180°, finally we translate it. The combination of reflection, rotation, and translation is called glide reflection as shown in figure 11. When you reflect a point across the line y = x, the x-coordinate, and the y-coordinate change places. We can see the effect of vertical reflection and 180° centre point rotation isometry on the (F) letter changes but on IGP design stays the same.

$$P\ (x, y) \rightarrow P'\ (y, x) \quad \text{or} \quad r_{\ y=x}\ (x, y) = (y, x)$$

Figure 11: The Effect of Frieze Vertical Reflection 180° Centre Point Rotation Symmetry on IGP

5. **Frieze Centre Point Rotation 180° and Translation**

This group of frieze symmetry consists of a rotation from its centre point by 180°, and then translation. The combination of rotation and translation is called glide reflection as shown in figure 12. When you reflect a point across the line y = -x, the x-coordinate and the y-coordinate change places and are negated (the signs are changed). We can see the effect of center point rotation 180° and translation isometry on the (F) letter changes but on IGP design stays the same.

$$P\ (x, y) \rightarrow P'\ (-y, -x) \quad \text{or} \quad r_{\ y=-x}\ (x, y) = (-y, -x)$$

Figure 12: The Effect of Frieze Centre Point Rotation 180° and Translation Symmetry on IGP

6. **Frieze Horizontal Reflection and Translation**

This group of frieze symmetry consists of horizontal reflection and then translation. The combination of horizontal reflection and translation is called glide reflection as shown in figure 13. Point reflection exists when a figure is built around a single point called the center of the figure. It is a direct isometry. Origin Reflections: while any point in the coordinate plane may be used as a point of reflection, the most used point is the origin. We can see the effect of horizontal reflection and translation isometry on the (F) letter changes but on IGP design stays the same.

$$P\ (x, y) \rightarrow P'\ (-x, -y) \quad \text{or} \quad r_{\ origin}\ (x, y) = (-x, -y)$$

Figure 13: The Effect of Frieze Horizontal Reflection and Translation Symmetry on IGP

7. **Frieze Corner Point and Middle Side Rotation**

This group of frieze symmetry consists of 180° rotational symmetry at the corner point, or the middle side as shown in figure 14 and 15. Assuming center of rotation to be the origin, a rotation turns the figure through an angle about a fixed point called the center. A positive angle of rotation turns the figure counterclockwise, and a negative angle of rotation turns the figure in a clockwise direction. It is a direct isometry. We can see the effect of rotational isometry on the (F) letter changes but on IGP design stays the same.

$$P_{90^\circ}(x, y) \rightarrow P'(-y, x) \quad P_{180^\circ}(x, y) \rightarrow P'(-x, -y) \quad P_{270^\circ}(x, y) \rightarrow P'(y, -x)$$

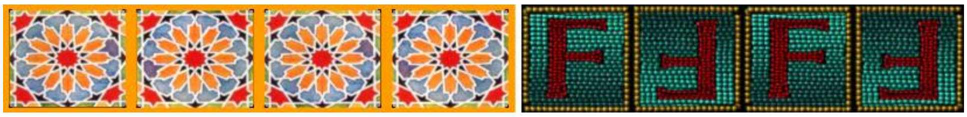


Figure 14: The Effect of Frieze Corner Point and Middle Side Rotation Symmetry on IGP

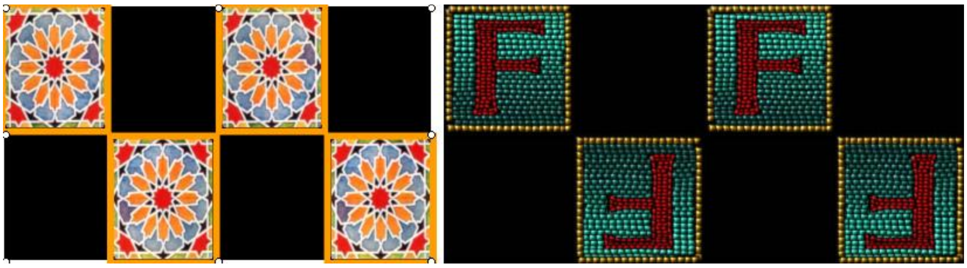


Figure 15: The Effect of Frieze Corner Point and Middle Side Rotation Symmetry on IGP

Wallpaper Group Classification

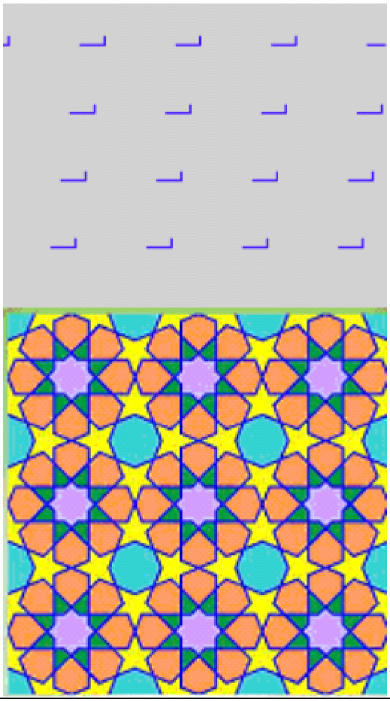
In 1924 that George Pólya showed that there are exactly 17 mathematically distinct ways of tiling or tessellating a surface [64]. In 1927 Andreas Speiser had called attention to Islamic art and considers the scientific theoretical finding and reveals the value of these scientific theories to the understating the classification of IGP [81]. It was only in 1935 that the scientific findings of point group theory were fixed in the international tables of crystallography. These notations became the most widely used language by the chemists. Crystallography has 230 space groups to distinguish, far more than the 17 wallpaper groups, but many of the symmetries in the groups are the same. A complete notation of all seventeen possible wallpaper groups is in the following table by the International Union of Crystallography in 1952 (IUC).

Edith Muller had written a thesis on group theory and symmetry notation of the patterns of Moorish ornaments in the Al-Hambra palace. He carried out a mathematical symmetry analysis of designs which shows that there are exactly 17 different plane symmetry groups [82]. Wallpaper groups categorize patterns by their symmetries. All 17 groups were used by Egyptian craftsmen and used extensively in the Muslim world [80]. Niman and Norman stated that the use of two-dimensional Euclidean geometry in artistic modes of expression was most pronounced in Islamic art. It constituted the basis for the formation of rhythmic and intertwining patterns, elegant calligraphic inscriptions, and blossoming two-dimensional crystallographic patterns [83]. Abas and Salman in their article categorized any two-dimensional periodic pattern into one of 17 types of wallpaper symmetry group theory. These 17 types are known as periodic groups of wallpaper groups or plane crystallographic groups [84]. Joyce reported that various planar patterns can be classified by transformation groups that leave them invariant. A mathematical analysis of these groups shows that there are exactly 17 different plane symmetry groups [85]. On the effects of symmetry on a specific wallpaper theory, a group of researchers explains that lines of reflection emphasize stability and rest. A line unimpeded by perpendicular reflections encourages movement. Rotational symmetries are also considered dynamic [65]. Patterns that are invariant under linear combinations of two linearly independent translations repeat at regular intervals in two directions, and hence their groups are often termed wallpaper groups [4]. A wallpaper group is a mathematical classification of a two-dimensional repetitive pattern, based on the symmetries in the pattern. Such patterns occur frequently in architecture and decorative art.

A mathematical analysis of these groups shows that there are exactly 17 different plane symmetry groups. All the 17 distinct types of the pattern have been used by craftsmen using tiles or weaving or decorating walls or any of the other myriad ways in which humans cover their everyday objects with patterns, imposing what Gombrich has called ‘the sense of order’. For wallpaper groups, the full notation begins with either (p) or (c), for a primitive cell. This is followed by a digit, (n), indicating the highest order of rotational symmetry: 1-fold (none), 2-fold, 3-fold, 4-fold, or 6-fold. The next two symbols indicate symmetries related to one translation axis of the pattern. The symbols are either (m), (g), or (1), for the mirror, glide reflection, or none [64]. Interestingly, although all the 17 types of the pattern can be found in art form around the world, not all types are found in all cultures. Although it is sometimes claimed that all the 17 types can be found in the Alhambra, it seems that only 13 of the types are there. Of the remaining four types it is said that two have been found elsewhere in Islamic art, but that the other two, specifically (pg) and (pgg), are not found anywhere in Islamic Art [68].

Here are visual representations of the 17-wallpaper group theory and their compression with IGP patterns in an attempt by the researcher to demonstrate that the current 17-wallpaper group theory is merely arrangement classification and not classification based on normalization grid methodology as shown in figures 16-32.

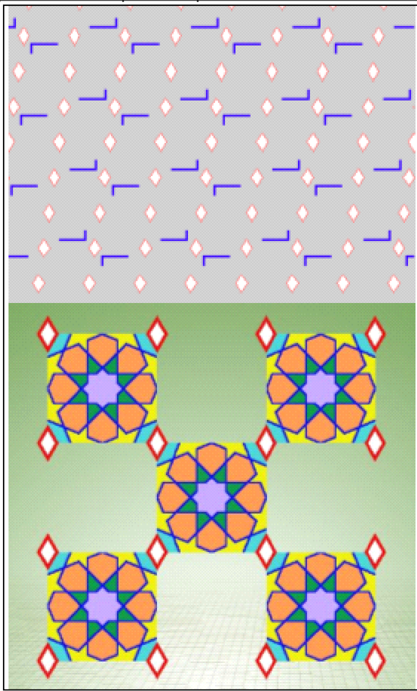
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p1	C_1	Aucun	$H_{f_{L_s},p1}(x) = \sum_{g \in c_1} f_{L_s}\,[g(x)]$



Wallpaper group 1 (p1) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	Rotation (90°)	X
4	Rotation (120°)	X
5	Rotation (180°)	X
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	Glide-Reflection (Horizontal)	X
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**1**

Figure 16: The Effect of Wallpaper (p1) Translation Symmetry on IGP

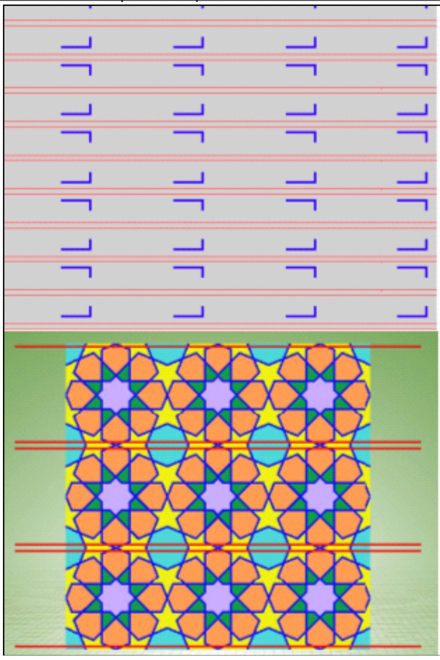
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p2	C_2	Aucun	$H_{f_{L_s},p2}(x) = \sum_{g \in c_2} f_{L_s}\,[g(x)]$



Wallpaper group 2 (p2) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	Rotation (90°)	X
4	Rotation (120°)	X
5	**Rotation (180°)**	√
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	Glide-Reflection (Horizontal)	X
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**2**

Figure 17: The Effect of Wallpaper (p2) Symmetry on IGP Design

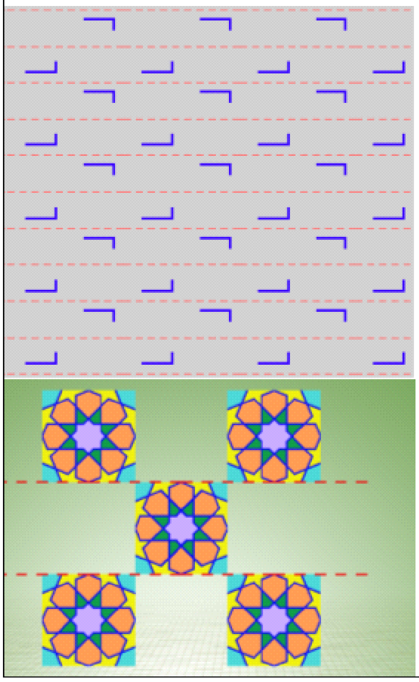
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
pm	D_1	$\sigma_1(a,b) = (a,-b)$	$H_{f_{L_s},pm}(x) = \sum_{g \in D_1} f_{L_s}\,[g(x)] + \sum_{g \in D_1} f_{L_s}\,[(\sigma_1 g)(x)]$



Wallpaper group 3 (pm) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	Rotation (90°)	X
4	Rotation (120°)	X
5	Rotation (180°)	X
6	**Regular-Reflection (Horizontal)**	√
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	Glide-Reflection (Horizontal)	X
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**2**

Figure 18: The Effect of Wallpaper (pm) Symmetry on IGP Design

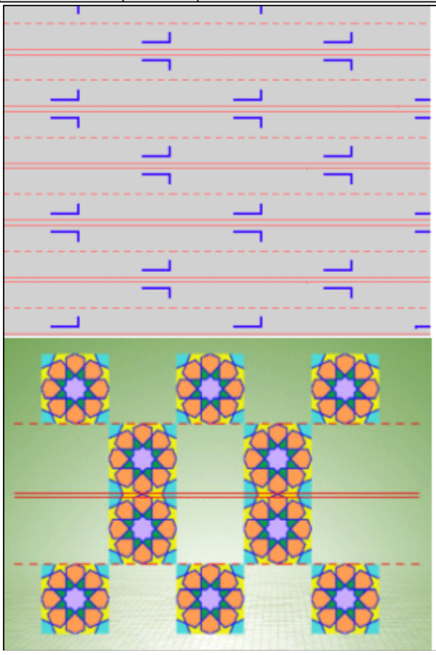
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
pg	D_1	$\sigma_1(a,b) = (\pi + a, -b)$	$H_{f_{L_s},pg}(x) = \sum_{g \in D_1} f_{L_s}[g(x)] + \sum_{g \in D_1} f_{L_s}[(\sigma_1 g)(x)]$



Wallpaper group 4 (pg) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	Rotation (90°)	X
4	Rotation (120°)	X
5	Rotation (180°)	X
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	**Glide-Reflection (Horizontal)**	√
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**2**

Figure 19: The Effect of Wallpaper (pg) Symmetry on IGP Design

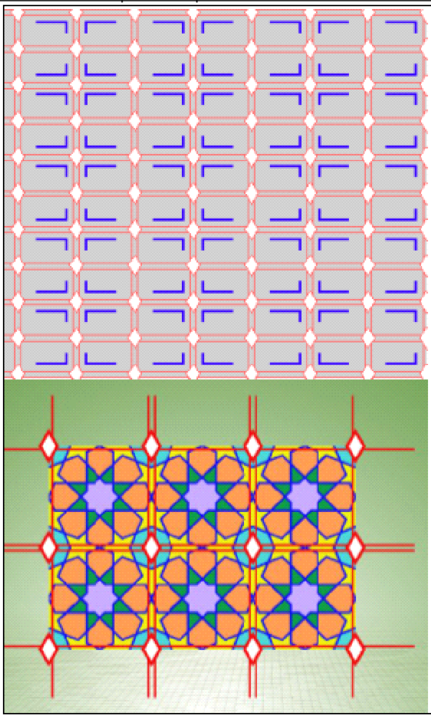
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
cm	D_1	$\sigma_1(a,b) = (a, -b)$ $\sigma_2(a,b) = (\pi + a, \pi - b)$	$H_{f_{L_s},cm}(x) = \sum_{g \in D_1} f_{L_s}[g(x)] + \sum_{i=1}^{2} \left\{ \sum_{g \in D_1} f_{L_s}[(\sigma_i g)(x)] \right\}$



Wallpaper group 5 (cm) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	Rotation (90°)	X
4	Rotation (120°)	X
5	Rotation (180°)	X
6	**Regular-Reflection (Horizontal)**	√
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	**Glide-Reflection (Horizontal)**	√
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**3**

Figure 20: The Effect of Wallpaper (cm) Symmetry on IGP Design

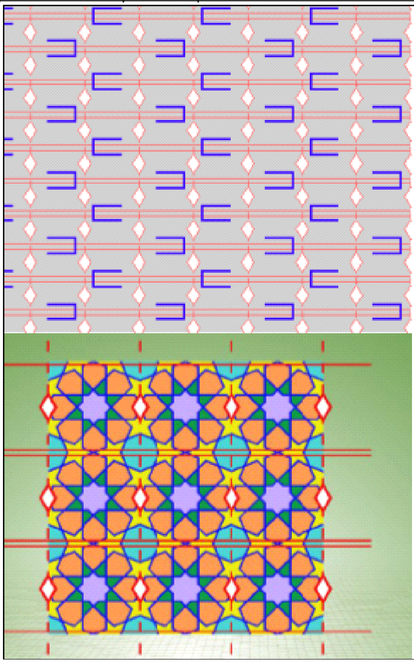
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
pmm	D_2	$\sigma_1(a,b) = (a,-b)$ $\sigma_2(a,b) = (-a,b)$	$H_{f_{L_s},pmm}(x) = \sum_{g\in D_2} f_{L_s}[g(x)] + \sum_{i=1}^{2}\left\{\sum_{g\in D_2} f_{L_s}\,[(\sigma_i g)(x)]\right\}$



Wallpaper group 6 (pmm) Operations		
1	**Translation**	√
2	Rotation (60º)	X
3	Rotation (90º)	X
4	Rotation (120º)	X
5	**Rotation (180º)**	√
6	**Regular-Reflection (Horizontal)**	√
7	**Regular-Reflection (Vertical)**	√
8	Regular-Reflection (30º)	X
9	Regular-Reflection (45º)	X
10	Regular-Reflection (60º)	X
11	Glide-Reflection (Horizontal)	X
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45º)	X
14	Glide-Reflection (60º)	X
Total Operation		**4**

Figure 21: The Effect of Wallpaper (pmm) Symmetry on IGP Design

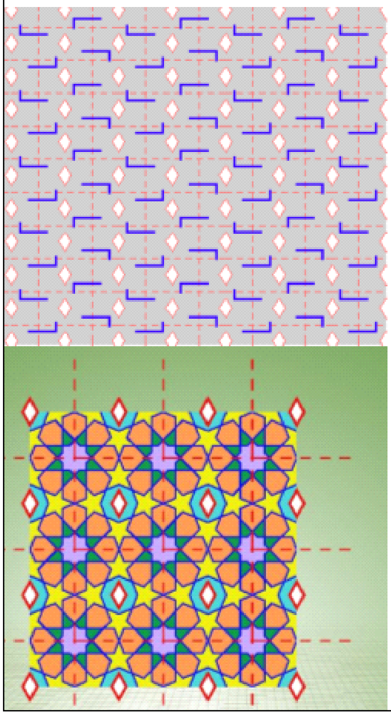
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
pmg	D_2	$\sigma_1(a,b) = (\pi + a,-b)$ $\sigma_2(a,b) = (-a,b)$	$H_{f_{L_s},pmg}(x) = \sum_{g\in D_2} f_{L_s}[g(x)] + \sum_{i=1}^{2}\left\{\sum_{g\in D_2} f_{L_s}\,[(\sigma_i g)(x)]\right\}$



Wallpaper group 7 (pmg) Operations		
1	**Translation**	√
2	Rotation (60º)	X
3	Rotation (90º)	X
4	Rotation (120º)	X
5	**Rotation (180º)**	√
6	**Regular-Reflection (Horizontal)**	√
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30º)	X
9	Regular-Reflection (45º)	X
10	Regular-Reflection (60º)	X
11	Glide-Reflection (Horizontal)	X
12	**Glide-Reflection (Vertical)**	√
13	Glide-Reflection (45º)	X
14	Glide-Reflection (60º)	X
Total Operation		**4**

Figure 22: The Effect of Wallpaper (pmg) Symmetry on IGP Design

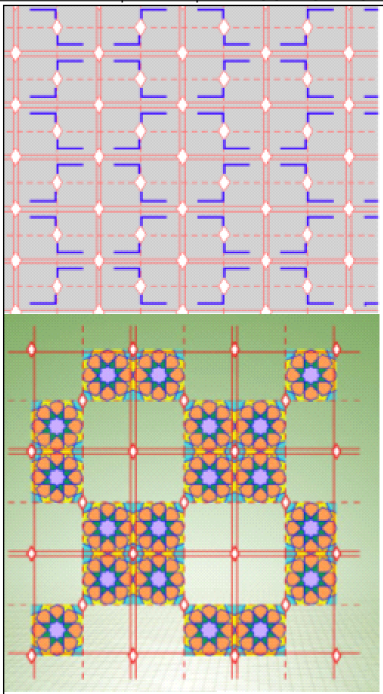
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
Pgg	D_2	$\sigma_1(a,b)=(\pi+a,\pi-b)$ $\sigma_2(a,b)=(\pi-a,\pi+b)$	$H_{f_{L_s},pgg}(x)=\sum_{g\in D_2} f_{L_s}[g(x)]+\sum_{i=1}^{2}\left\{\sum_{g\in D_2} f_{L_s}\,[(\sigma_i g)(x)]\right\}$



Wallpaper group 8 (pgg) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	Rotation (90°)	X
4	Rotation (120°)	X
5	**Rotation (180°)**	√
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	**Glide-Reflection (Horizontal)**	√
12	**Glide-Reflection (Vertical)**	√
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**4**

Figure 23: The Effect of Wallpaper (pgg) Symmetry on IGP Design

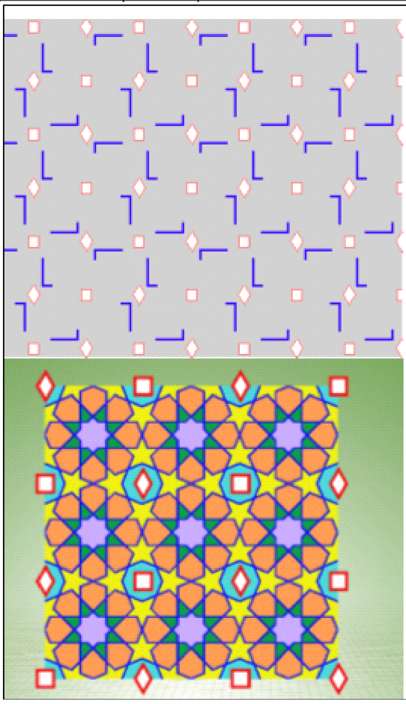
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
cmm	D_2	$\sigma_1(a,b)=(a,-b)$ $\sigma_2(a,b)=(\pi-a,\pi+b)$ $\sigma_3(a,b)=(\pi+a,\pi-b)$ $\sigma_4(a,b)=(-a,b)$	$H_{f_{L_s},cmm}(x)=\sum_{g\in D_2} f_{L_s}[g(x)]+\sum_{i=1}^{4}\left\{\sum_{g\in D_2} f_{L_s}\,[(\sigma_i g)(x)]\right\}$



Wallpaper group 9 (cmm) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	Rotation (90°)	X
4	Rotation (120°)	X
5	**Rotation (180°)**	√
6	**Regular-Reflection (Horizontal)**	√
7	**Regular-Reflection (Vertical)**	√
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	**Glide-Reflection (Horizontal)**	√
12	**Glide-Reflection (Vertical)**	√
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**6**

Figure 24: The Effect of Wallpaper (cmm) Symmetry on IGP Design

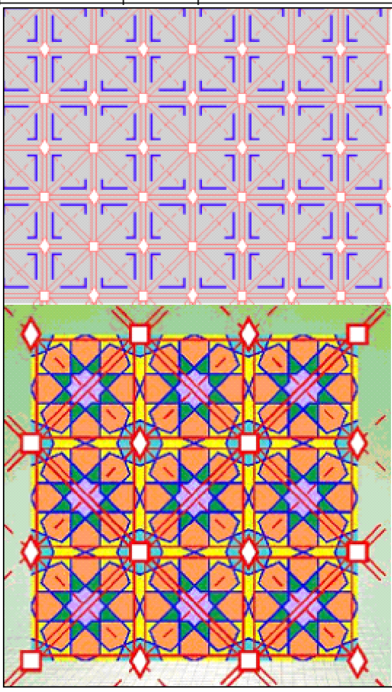
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p4	C_4	Aucun	$H_{f_{L_s},p4}(x) = \sum_{g \in C_4} f_{L_s}\ [g(x)]$



Wallpaper group 10 (p4) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	**Rotation (90°)**	√
4	Rotation (120°)	X
5	**Rotation (180°)**	√
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	Glide-Reflection (Horizontal)	X
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**3**

Figure 25: The Effect of Wallpaper (p4) Symmetry on IGP Design

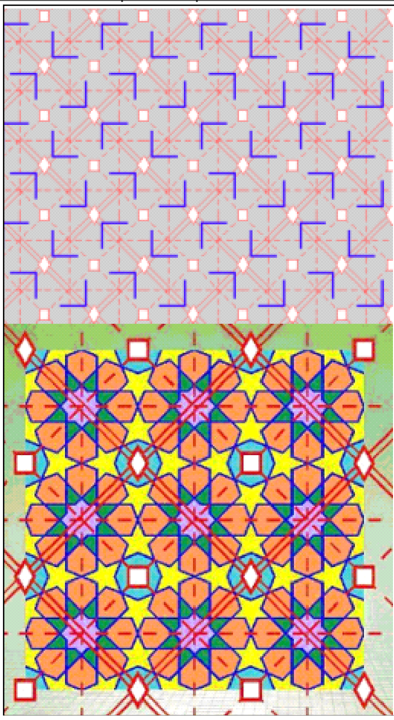
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p4m	D_4	$\sigma_1(a,b) = (a,-b)$	$H_{f_{L_s},p4m}(x) = \sum_{g \in D_4} f_{L_s}\ [g(x)] + \sum_{g \in D_4} f_{L_{ds}}\ [(\sigma_1 g)(x)]$



Wallpaper group 11 (p4m) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	**Rotation (90°)**	√
4	Rotation (120°)	X
5	**Rotation (180°)**	√
6	**Regular-Reflection (Horizontal)**	√
7	**Regular-Reflection (Vertical)**	√
8	Regular-Reflection (30°)	X
9	**Regular-Reflection (45°)**	√
10	Regular-Reflection (60°)	X
11	Glide-Reflection (Horizontal)	X
12	Glide-Reflection (Vertical)	X
13	**Glide-Reflection (45°)**	√
14	Glide-Reflection (60°)	X
Total Operation		**7**

Figure 26: The Effect of Wallpaper (p4m) Symmetry on IGP Design

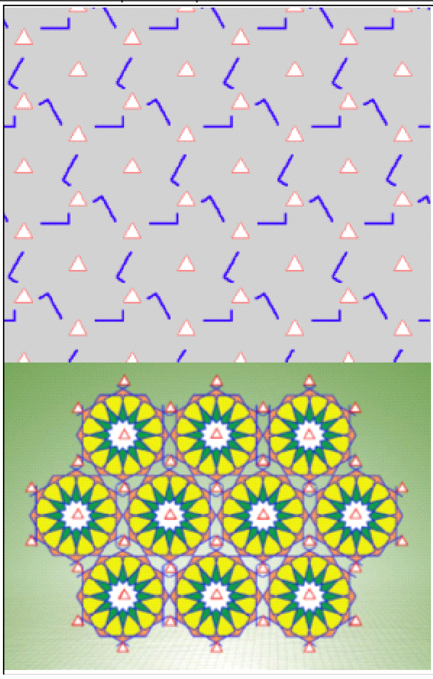
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p4g	D_4	$\sigma_1(a,b) = (\pi + a, -b)$	$H_{f_{L_s},p4g}(x) = \sum_{g \in D_4} f_{L_s}[g(x)] + \sum_{g \in D_4} f_{L_s}[(\sigma_1 g)(x)]$



Wallpaper group 12 (p4g) Operations		
1	**Translation**	√
2	Rotation (60º)	X
3	**Rotation (90º)**	√
4	Rotation (120º)	X
5	**Rotation (180º)**	√
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30º)	X
9	**Regular-Reflection (45º)**	√
10	Regular-Reflection (60º)	X
11	**Glide-Reflection (Horizontal)**	√
12	**Glide-Reflection (Vertical)**	√
13	**Glide-Reflection (45º)**	√
14	Glide-Reflection (60º)	X
Total Operation		**7**

Figure 27: The Effect of Wallpaper (p4g) Symmetry on IGP Design

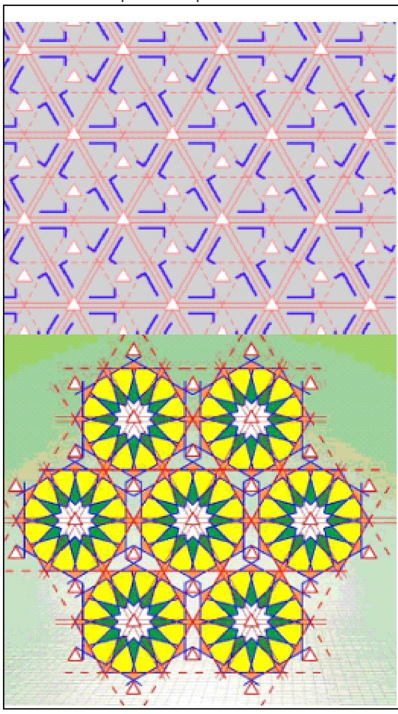
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p3	C_3	Aucun	$H_{f_{L_d},p3}(x) = \sum_{g \in c_3} f_{L_d}[g(x)]$



Wallpaper group 13 (p3) Operations		
1	**Translation**	√
2	Rotation (60º)	X
3	Rotation (90º)	X
4	**Rotation (120º)**	√
5	Rotation (180º)	X
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30º)	X
9	Regular-Reflection (45º)	X
10	Regular-Reflection (60º)	X
11	Glide-Reflection (Horizontal)	X
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45º)	X
14	Glide-Reflection (60º)	X
Total Operation		**2**

Figure 28: The Effect of Wallpaper (p3) Symmetry on IGP Design

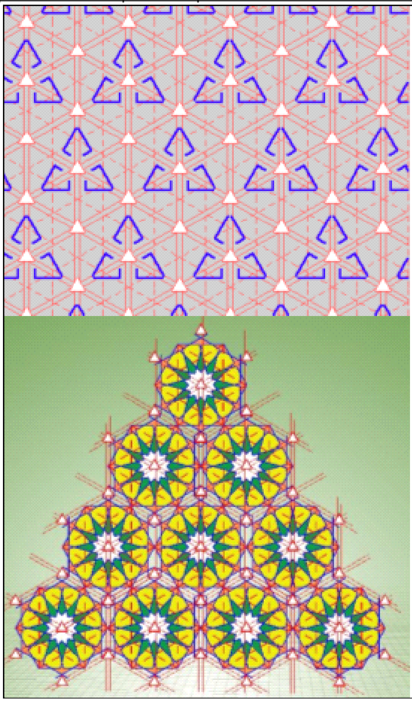
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p31m	D_3	$\sigma_1(a,b) = (a,-b)$	$H_{f_{L_d},p31m}(x) = \sum_{g \in D_3} f_{L_d}[g(x)] + \sum_{g \in D_3} f_{L_d}[(\sigma_1 g)(x)]$



Wallpaper group 14 (p3lm) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	Rotation (90°)	X
4	**Rotation (120°)**	√
5	Rotation (180°)	X
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	**Regular-Reflection (60°)**	√
11	**Glide-Reflection (Horizontal)**	√
12	Glide-Reflection (Vertical)	X
13	**Glide-Reflection (45°)**	√
14	Glide-Reflection (60°)	X
Total Operation		**5**

Figure 29: The Effect of Wallpaper (p3lm) Symmetry on IGP Design

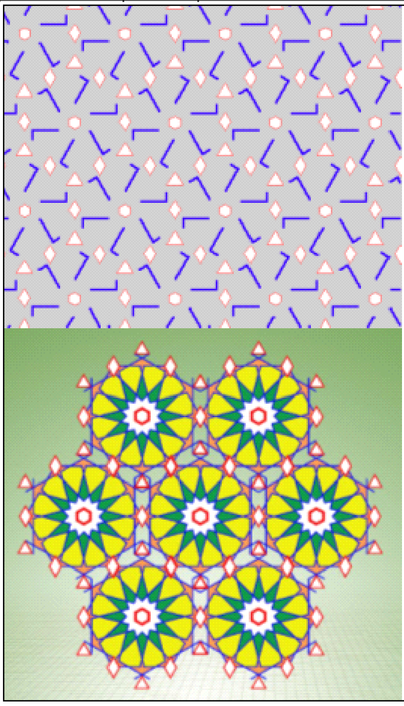
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p3m1	D_3	$\sigma_1(a,b) = (-a,b)$	$H_{f_{L_d},p3m1}(x) = \sum_{g \in D_3} f_{L_d}[g(x)] + \sum_{g \in D_3} f_{L_d}[(\sigma_1 g)(x)]$



Wallpaper group 15 (p3ml) Operations		
1	**Translation**	√
2	Rotation (60°)	X
3	Rotation (90°)	X
4	**Rotation (120°)**	√
5	Rotation (180°)	X
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	**Regular-Reflection (60°)**	√
11	Glide-Reflection (Horizontal)	X
12	**Glide-Reflection (Vertical)**	√
13	**Glide-Reflection (45°)**	√
14	Glide-Reflection (60°)	X
Total Operation		**5**

Figure 30: The Effect of Wallpaper (p3ml) Symmetry on IGP Design

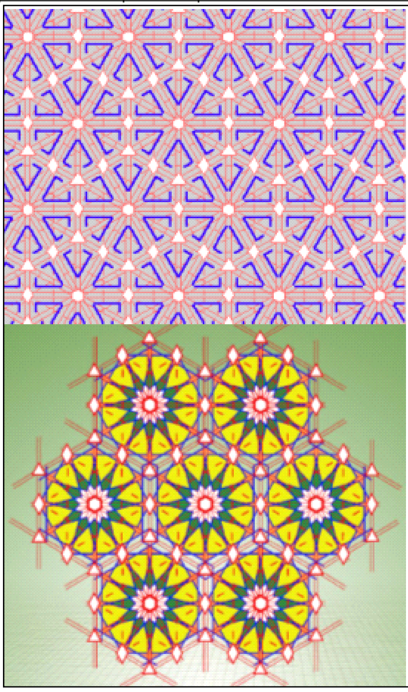
Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p6	C_6	Aucun	$H_{f_{L_d},p6}(x) = \sum_{g\in C_6} f_{L_d}\,[g(x)]$



Wallpaper group 16 (p6) Operations		
1	**Translation**	√
2	**Rotation (60°)**	√
3	Rotation (90°)	X
4	**Rotation (120°)**	√
5	**Rotation (180°)**	√
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	Regular-Reflection (30°)	X
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	Glide-Reflection (Horizontal)	X
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**4**

Figure 31: The Effect of Wallpaper (p6) Symmetry on IGP Design

Wallpaper group	Point group	Extra symmetry set	Invariant mapping
p6m	D_6	$\sigma_1(a,b) = (a,-b)$	$H_{f_{L_d},p6m}(x) = \sum_{g\in D_6} f_{L_d}\,[g(x)] + \sum_{g\in D_6} f_{L_d}\,[(\sigma_1 g)(x)]$



Wallpaper group 17 (p6m) Operations		
1	**Translation**	√
2	**Rotation (60°)**	√
3	Rotation (90°)	X
4	**Rotation (120°)**	√
5	**Rotation (180°)**	√
6	Regular-Reflection (Horizontal)	X
7	Regular-Reflection (Vertical)	X
8	**Regular-Reflection (30°)**	√
9	Regular-Reflection (45°)	X
10	Regular-Reflection (60°)	X
11	Glide-Reflection (Horizontal)	X
12	Glide-Reflection (Vertical)	X
13	Glide-Reflection (45°)	X
14	Glide-Reflection (60°)	X
Total Operation		**5**

Figure 32: The Effect of Wallpaper (p6m) Symmetry on IGP Design

Design Methodologies

Islamic geometric patterns led to several significant design methodologies that advocate the design in different ways. The study of IGP over centuries has had various dimensions and was viewed with varied ranging prisms. However, experts like Abas and Salman studied them under the umbrella of tessellation primitive techniques. They suggest the practical experience of tiling and covering with simple naturally occurring shapes as an original methodology which would seem most logical. The shapes would then be worked on giving rise to triangles, rectangles, squares, hexagons. The shapes would have been decorated with simple colours and patterns. From this beginning, the next stage would be to experiment with multiple shaped tiles, with shapes produced by overlapping tiles [84]. Issam El-Said portrays a historical dimension to the origin of IGP and their developments. He noted that the absolute mastery of mathematics combined with the rhythmic repetitions and symmetric arts which they gained from the Byzantine and Sassanid empires resulted in a blasting artistic expansion [17]. It is a common knowledge that Islamic Art has imbibed the characteristic features of the native regions into which spread over several hundreds of years. Islamic geometric patterns by their very native nature present themselves to be explored by using a grid system. A grid is a basic frame subjective to each design. The design itself might usually be formed by placing primary and secondary elements of the design on a basic grid. A grid is a very powerful visual layout that presents attributes to the design as shown in figures 33-41:

Square Grid Design Methodology

One of the most fundamental and popular grids is the square grid. It is most extensively used by traditional designers and is used as the fundamental building block of raster displays as shown in figure 33.

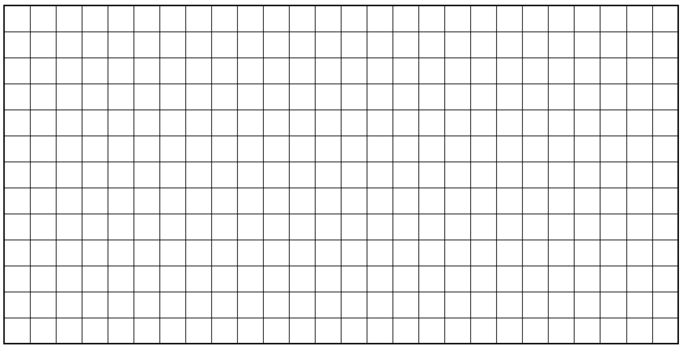


Figure 33: Square Grid Design Methodology

Calligraphy Design Methodology

Islamic calligraphers have used the square grid as the basic slate for displaying the holy revelations of the Quran and from time immemorial these grids have facilitated stunning writing of beautifully decorated verses of the sacred text on them as shown in figure 34.

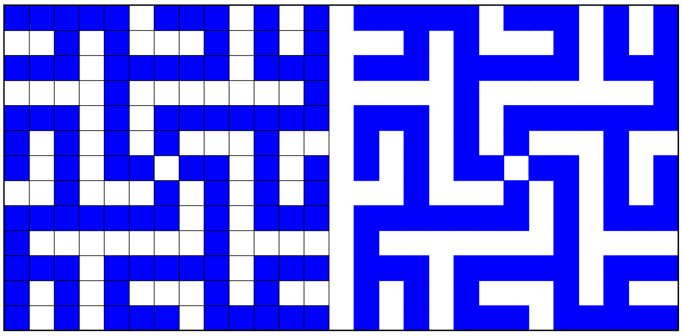


Figure 34: Calligraphic Design Methodology

Interlocking Design Methodology

Interlocking design is another adaptation of a square grid. A collection of square grids can be amalgamated or merged to generate a geometric pattern design. But even if the design deepest significance eluded the designers, experts say that only people with an advanced grasp of geometry could have made such elaborate, interlocking designs as shown in figure 35.

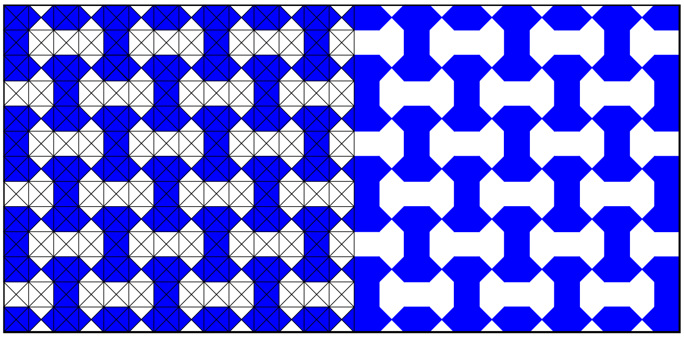


Figure 35: Interlocking Design Methodology

Triangulation Design Methodology

Another extension of the square grid is a simple triangulation usually achieved by overlapping two grids tilted at 45 degrees and matching their nodes. The added layer usually generates at least a minimum of four-fold complex design as shown in figure 36.

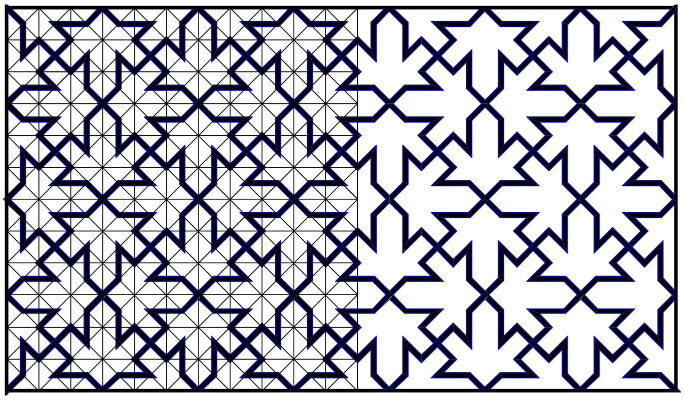


Figure 36: Triangulation Design Methodology

Isometric Design Methodology

When this triangular grid is further explored it matures into isometric shapes which are a very popular form of stone artwork under the aegis of Islamic art as shown in figure 37.

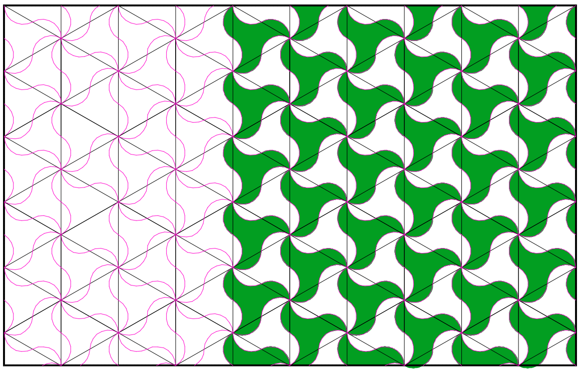


Figure 37: Isometric Design Methodology

Overlapped Design Methodology

A further exploration in this series is achieved by combining the horizontal and vertical line spacing to produce a design with overlapping effects as shown in figure 38.

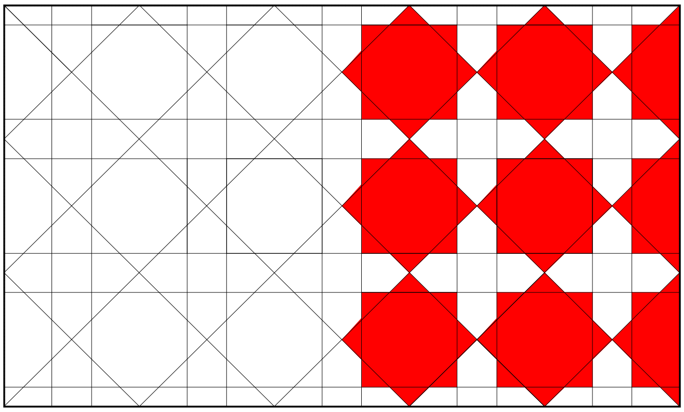


Figure 38: Overlapped Design Methodology

Radial / Bilateral Design Methodology

Overlapping can result in designs with absolute complexity. E. H. Hankin [136] has analysed a complex pattern in the year 1925 and referred to it personally as A Difficult Saracenic Design, which he further referred to the designs as “Saracenic design each having pattern space, either radial or bilateral symmetry” as shown in figure 39.

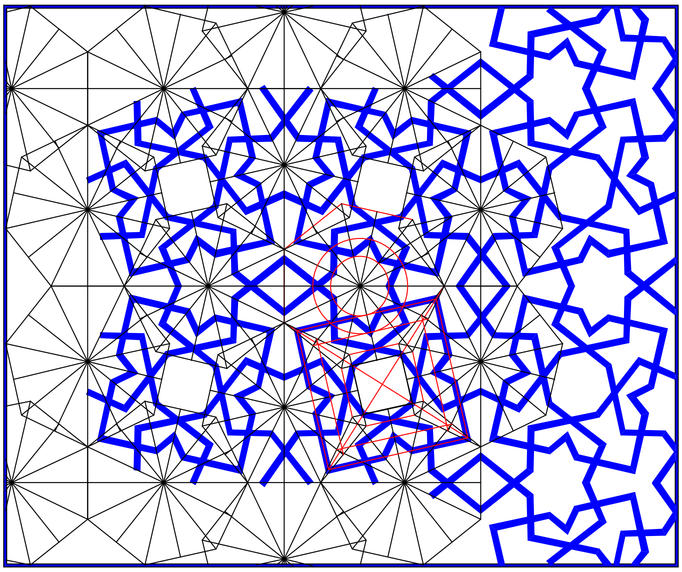


Figure 39: Radial/Bilateral Design Methodology

Ruler Compass Design Methodology

In the field of IGP, the ruler/compass method is associated with Issam El-Said [17]. His obsession with geometry started over five decades ago when he began to formulate the IGP grid system concepts determining the formulae used by craftsmen and master builders in Islamic art and architecture which although was never recorded as a design methodology. His methodology of composition and system of measure avoids complicated mathematical calculations, rather an artist can create an IGP design with the use of simple tools. The inherent grid system and geometric proportions, which he pursued in his designs, paintings, and research, were regarded by him-self as the conceptual essence of balance (al-Mizan) mentioned in the Holy Quran:

“*And the heaven He raised and imposed the balance*”, [60, 55:7]. وَٱلسَّمَآءَ رَفَعَهَا وَوَضَعَ ٱلْمِيزَانَ ۝٧

His research led him to investigate the foundations and principles that he later identified as a principle (*Usul*), which he conceived as the system of creativity in the IGP artistic heritage. They also believe that the overall impression that is created is that from the earliest of times the inventors of IGP designs were inspired and dedicated geometers by theoretical ruler/compass-based construction of the classical Greek geometry as normal concatenate step forward toward the IGP designs as shown in figure 40.

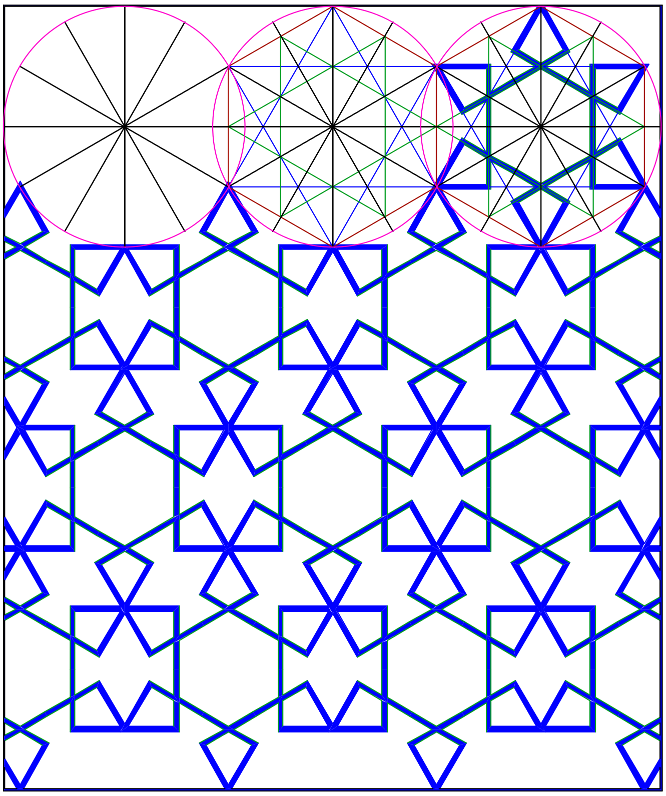


Figure 40: Ruler Compass Design Methodology

Numerical Design Methodology

Salman has done pioneering work in this method wherein he utilized the state of a software that has facilitated methods that generate IGP based on numerical inputs extracted from the attributes of the IGP motif. His software demonstrated the seamless processing of numerical inputs to result in tangible outputs [137]. He has produced an array of self-descriptive 17-wallpaper group images using this digital technique, which can be used to conjoin one a IGP motif with another to form the final design through mathematical means as shown in figure 41.

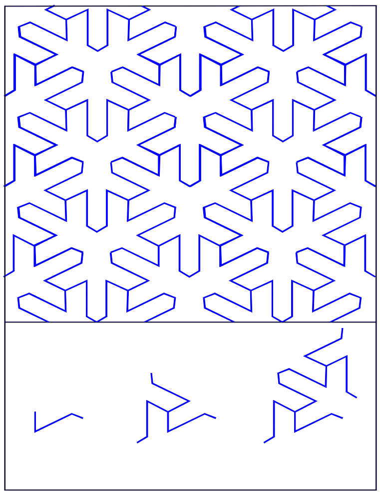


Figure 41: Numerical Design Methodology

Conclusion

The two ancient Sufi texts by Al-Boni and Al-Ahsai are neither the works of any author of the current age nor of any university department. Ancient experts had written them several hundreds of years ago. Both these Sufi texts are important watersheds in deciphering the sacred meanings behind designs. They cover the relationship and potential metaphysical power between designs and sacred meanings, numerals, geometry, celestial bodies, and alphabets. Also, clarify the thought process of the designers of those ages. The foreword written by Nasr in Critchlow's book suggest that; "*within the spiritual universe of Islam there is a dimension which is called Abrahamic Pythagoreanism, or a way of seeing numbers and figures as keys to the structure of the cosmos and as symbols of the archetypal world and also a world which is viewed as the creation of God in the sense of Abrahamic monotheism*". Moreover, Critchlow suggests that Islamic art is gradually coming to be understood for what it is, namely a means of relating multiplicity to Unity using mathematical forms which are seen, not as mental abstractions, but as reflections of the celestial archetypes within both the cosmos and the minds and souls of men [13].

One of the primary purposes of this thesis is to present an innovative methodology to classify and design IGP designs based on a normalisation grid methodology. Therefore, in this thesis, the researcher uses symmetry to refer more generally to a mathematical theory that accounts for the repetition in a style of design. The normalization grid methodology proposed in this thesis suggests classification for IGP based on grid system formation dictated by the minimum number of grids and the lowest geometrical shape used in designs. Conventional design studies have merely focused on the base principles of ruler/compass methodology. This thesis attempts to do away with the hierarchical chronological order of the formation of IGP with lesser and more prudent number of tangible forms that are bounded by sheer balance its symmetry and motif segment that literally hold the entire form of a design. This thesis stresses on the absolute importance of symmetry in visualizing the balance and equilibrium of normalisation grid methodology within the design. It demonstrates to the reader how a mathematical approach to the motif with the aid of software gives us extra resources to do our research.

It is clear from the above illustrations that the 7-frieze and 17-wallpaper group theories enable us only to classify the type of arrangements for an asymmetry design and not suitable for IGP symmetrical designs as been demonstrated previously. There has been no viable approach to classify IGP and to fill this vacuum, the researcher has proposed the normalization grid methodology for IGP classification. The revelation of this thesis would not only offer a precious resource for art communities, but also it would be of interest to mathematicians, crystallographers, architects, archaeologists, fine arts, jewellery, tattooing, sculptures, and others to explore the use of IGP in their artwork. They can serve as elegant testbeds for research into hierarchical programming and texture mapping. The work has scanned through all the literature available on IGP, right from its foundation through the gradual development up to the present state of digitalising through mathematical computer programs. This thesis has the unique peculiarity that it is trying to correlate the ancient philosophical revelations with the modern developments in the field of IGP.

Chapter 3: Program Design

The objective of this thesis is to develop, test and evaluate the IGP software that enables to classify and design designs. The aim of this process is to construct accurate two-dimensional models of IGP designs, classify, generate motifs, modify motif parameters for new creative designs. This chapter addresses the development process of the IGP software in two phases: the conceptual design phase and the implementation design phase.

System Design Analysis

Software design is the preliminary step and is also a building block of software engineering. The efficiency of the software is promoted through design phase. The design phase begins when the requirement specification document for the software to be developed is available. While the requirement specification activity is entirely in the problem domain design step is the step to moving from the problem domain towards the solution domain. Design is the bridge between requirement specification and the final solution for satisfying the requirements. The term design is used in two ways. User as a verb represents the process of design. User as a noun, it represents the result of design process, it is the design for the system. The goal of the design process is to produce a model or the representation of a system, which can be used later to build the system. The produced model is called the design of the system. System design is a process of evaluating alternative solution, evaluating the choice following up the specification for the chosen alternative. System design is to improve the existing system or designing a new system and implement the system with improved facilities.

Existing System

The IGP in most cases is designed based on ruler/compass methodology, which the researcher considers as general. The challenge of how mathematically to find the parameters of the design motif which is based on the relationships embedded in the grid system to discover new IGP designs. Nevertheless, there are no standard methods for defining the parameters of IGP classification and design. When designing such a method the only restrictions are imposed by computer programs themselves, common sense and learned rules.

Proposed System

A successful software to integrate the mathematical and algorithmic rules of designs with visual environment techniques using computer tools is to develop a software capable of reading the normalization grid methodology to classify, generate motifs, modify motif parameters for new creative designs. Its main hypothesis is that this software offers many advantages over human power in terms of recognizing the necessary parameters for accurate classification and design digitally for further variety. It also has a great capacity for testing many designs quickly, accurately, and easily compared to the limitations of human capabilities. System requirement and specification:

1. Software Requirements: Operating System; Windows 7, Tool; Microsoft Visual Studio 2005, Language; C#.NET 2.0.
2. Hardware Requirements: Processor; Pentium IV OR Above, Primary Memory; 1 GB RAM, Storage; 320 GB Hard Disk, Monitor; SVGA color, Display; VGA Color Monitor, Keyboard; Multimedia Keyboard.

System Design Feasibility Study

All projects are feasible when given unlimited resources and infinite time. It is both necessary and prudent to evaluate the feasibility of a project at the earliest possible time. A feasibility study is not warranted for systems in which economic justification is obvious, technical risk is low, few legal problems are expected, and no reasonable alternative exists. An estimate is made of whether the identified user needs may be satisfied using current software and hardware technologies. The study will decide if the proposed system will be cost effective from the business point of view and if it can be developed in the given existing budgetary constraints. The feasibility study should be relatively cheap and quick. The result should inform the decision of whether to go ahead with a more detailed analysis. The feasibility study of has been conducted and found that the work can be completed in limited time and resources.

Technical Feasibility

The system must be evaluated from the technical viewpoint first. The assessment of this feasibility must be based on an outline design of the system requirement in terms of input, output, programs, procedure, and staff having identified the outline system. The investigation must go on to suggest the type of equipment required for running the system. This system has been developed using C#.NET, a popular GUI, which is very user friendly. Technical feasibility study deals with the hardware as well as software requirements. The scope was whether the work for the project is done with the current equipment and the existing software technology must be examined in the feasibility study. The outcome was found to be positive. To build this application it does require additional manpower and machine availability.

Economic Feasibility

One of the factors which affect the development of a new system is the cost it would require. Since the system is developed as a part of project work, there is no manual cost to be spent for the proposed system. Also, it requires minimum hardware. The justification of any capital outlay is that it will reduce expenditure or improve the quality of service or goods, which in turn may be expected to provide increased profit. The technique of cost benefit analysis is often used as a base for assessing economic feasibility. The cost of software implementation of this application is very less while comparing to the benefits.

Operational Feasibility

Proposed objects are beneficial only if they can be turned into information systems that will meet the organization's operating requirements. The purpose of the operational feasibility study is to determine whether there will be resistance from users that will underline the possible application benefits. If the users of the systems are fully aware of the internal working of the system, then the users will not be facing any problem in the running system. The software is designed for operational feasibility.

Behavioural Feasibility

People are inherently resistant to changes; but computers can and will be facilitating changes. An estimate should be made of how strongly the user staff reacts towards the development of the new computerized system. The existing system requires more manpower and more time. In the proposed system, both manpower and time factors are reduced. Thus, the remaining people can be deployed to engage in some other productive work. So, the system is behaviorally feasible. From all these studies, it can be concluded that the system is highly feasible.

Conceptual System Design

This is the initial phase of research and involves the intellectual process of developing the basic functions, operational characteristics, and the overall organization of tasks. This phase also includes designing the software interface, operation tasks, flow chart and investigating algorithmic approaches to narrowing down the topic until a succinct research problem and purpose have been determined and workable techniques identified. Since many of the tasks involved developing specialized algorithms and making use of graphics libraries, implementation required the researcher to have a strong computer graphics background which he experiences from his master's degree in computer animation and visualization, analytical skills, and general designing skills from his background as an architect. The programming languages used in this research are designed to communicate instructions and to express its algorithms precisely. The software executes a sequence of instructions to perform the task of classification and design. The proposed system is developed using Microsoft .NET Framework in C# as a desktop standalone application as shown in figure 42.

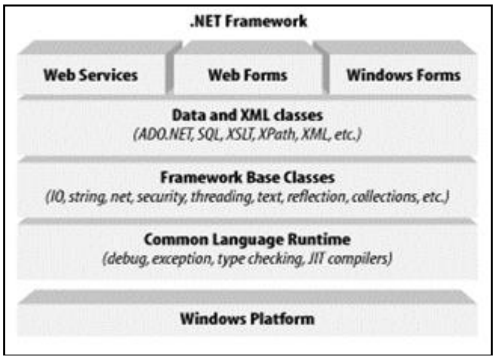


Figure 42: (.NET) Framework Architecture

1. **(C#) and the (.NET) Framework**

The goal of (C#) is to provide a simple, safe, modern, object-oriented, Internet-centric, high-performance language for (.NET) development. (C#) is a new language, but it draws on the lessons learned over the past three decades. One can easily see in (C#) the influence of Java, ((C++)), Visual Basic (VB), and other languages. It is a multi-paradigm programming language encompassing strong typing, imperative, declarative, functional, procedural, generic, object-oriented (class-based), and component-oriented programming disciplines. It was developed by Microsoft within its .NET initiative and later approved as a standard by Ecma (ECMA-334) and ISO (ISO/IEC 23270:2006). C# is one of the programming languages designed for the Common Language Infrastructure. C# is intended to be a simple, modern, general-purpose, object-oriented programming language.

2. **The (.NET) Platform**

The (.NET) platform of Microsoft is, in essence, a new development framework that provides a fresh application programming interface (API) to the services and APIs of classic Windows operating systems (especially the Windows 2000 family), while bringing together several disparate technologies that emerged from Microsoft during the late 1990s. Microsoft (.NET) supports not only language independence, but also language integration. It means that one can inherit from classes, catch exceptions, and take advantage of polymorphism across different languages. The (.NET) Framework makes this possible with a specification called the Common Type System (CTS) that all (.NET) components must obey. For example, everything in (.NET) is an object of a specific class that derives from the root class called System Object. The (CTS) supports the general concept of classes, interfaces, delegates (which support callbacks), reference types, and value types. Additionally, (.NET) includes a Common Language Specification (CLS), which provides a series of basic rules that are required for language integration. The (CLS) determines the minimum requirements for being a (.NET) language. Compilers that conform to the (CLS) create objects that can interoperate with one another. The entire Framework Class Library (FCL) can be used in any language that conforms to the (CLS). The (.NET) Framework sits on top of the operating system, which can be any flavor of Windows, and consists of several components. Currently, the (.NET) Framework consists of four official languages: (C#), (VB.NET), (Managed C++), and (JScript.NET). The Common Language Runtime (CLR), an object-oriented platform for Windows and web development that all these languages share several related class libraries, collectively known as the Framework Class Library (FCL). Figure 43 breaks down the (.NET) Framework into its system architectural components. The most important component of the (.NET) Framework is the (CLR), which provides the environment in which programs are executed. The (CLR) includes a virtual machine, objects, performs security checks on them, lays them out in memory, executes them, and garbage-collects them. (The Common Type System is also part of the CLR). In Figure 5.1, the layer on top of the (CLR) is a set of framework base classes, followed by an additional layer of data and XML classes, plus another layer of classes intended for web services, Web Forms, and Windows Forms. Collectively, these classes are known as the Framework Class Library (FCL), one of the largest class libraries in history and one that provides an object-oriented API to all the functionality that the (.NET) platform encapsulates. With more than 4,000 classes, the (FCL) facilitates rapid development of desktop, client/server, and other web services and applications. The set of framework base classes, the lowest level of the (FCL), is like the set of classes in Java. These classes support rudimentary input and output, string manipulation, security management, network communication, thread management, text manipulation, reflection, and collections functionality, etc. Windows Forms allow you to apply Rapid Application Development techniques to building web and Windows applications. Simply drag and drop controls onto your form, double-click control, and write the code to respond to the associated event.

3. Compilation and the MSIL

In (.NET), programs are not compiled into executable files; they are compiled into Microsoft Intermediate Language (MSIL) files, which the (CLR) then executes. The (MSIL) often shortened to (IL) files that (C#) produce is identical to the (IL) files that other (.NET) languages produce; the platform is language-agnostic. A key fact about the (CLR) is that it is common; the same runtime supports development in (C#) as well as in (VB.NET). (C#) code is compiled into IL when you build your project. The IL is saved in a file on disk. When you run your program, the (IL) is compiled again, using the Just-In-Time (JIT) compiler, a process often called (JIT'ing). The result is machine code, executed by the machine's processor. The standard (JIT) compiler runs on demand. When a method is called, the (JIT) compiler analyzes the IL and produces highly efficient machine code, which runs very fast. The (JIT) compiler is smart enough to recognize when the code has already been compiled, so as the application runs, compilation happens only as where needed. As (.NET) applications run, they tend to become faster and faster, as the already compiled code is reused. The (CLS) means that all (.NET) languages produce very similar (IL) code. As a result, objects created in one language can be accessed and derived from another. Thus, it is possible to create a base class in (VB.NET) and derive from it in (C#).

4. The (C#) Language

The (C#) language is disarmingly simple, with only about 80 keywords and a dozen built-in data types, but (C#) is highly expressive when it comes to implementing modern programming concepts. (C#) includes all the support for structured, component-based, object-oriented programming that one expects of a modern language built on the shoulders of (C++) and Java. The (C#) language was developed by a small team led by two distinguished Microsoft engineers, Anders Hejlsberg, and Scott Wiltamuth. Hejlsberg is also known for creating Turbo Pascal, a popular language for PC programming, and for leading the team that designed Borland Delphi, one of the first successful integrated development environments for client/server programming. At the heart of any object-oriented language is its support for defining and working with classes. Classes define new types, allowing you to extend the language to better model the problem you are trying to solve. (C#) contains keywords for declaring new classes and their methods and properties, and for implementing encapsulation, inheritance, and polymorphism, the three pillars of object-oriented programming. (C#) class definitions do not require separate header files or Interface Definition Language (IDL) files. Moreover, (C#) supports a new (XML) style of inline documentation that greatly simplifies the creation of online and print reference documentation for an application. (C#) also supports interfaces, a means of making a contract with a class for services that the interface stipulates. In (C#), a class can inherit from only a single parent, but a class can implement multiple interfaces. When it implements an interface, a (C#) class in effect promises to provide the functionality the interface specifies. (C#) also provides support for structs and component-oriented features, such as properties, events, and declarative constructs (called attributes). Component-oriented programming is supported by the (CLR's) support for storing metadata with the code for the class. An assembly is a collection of files that appear to the programmer to be a single dynamic link library (DLL) or executable (EXE). In (.NET), an assembly is the basic unit of reuse, versioning, security, and deployment. The (CLR) provides several classes for manipulating assemblies. A final note about (C#) is that it also provides support for directly accessing memory using (C++) style pointers and keywords for bracketing such operations as unsafe, and for warning the (CLR) garbage collector not to collect objects referenced by pointers until they are released.

5. (C#) versus (Visual Basic.NET)

The premise of the (.NET) Framework is that all languages are created equal. (C#) is an excellent language for (.NET) development. It is an extremely versatile, robust, and well-designed language. It is also currently the language most often used in articles and tutorials for (.NET) programming. It is likely that many (VB) programmers will choose to learn (C#), rather than upgrading their skills to (VB.NET). This would not be surprising because the transition from (VB6) to (VB.NET) is, arguably, nearly as difficult as from VB6 to (C#) and whether it's fair or not, historically; C-family programmers have had higher earning potential than (VB) programmers. As a practical matter, (C#) offers a wonderful chance to make a potentially lucrative transition.

6. **(C#) Versus Java**

   Java Programmers may look at (C#) with a mixture of trepidation, glee, and resentment. (C#) offers an easy transition for Java programmers; the syntax is very similar, and the semantics are familiar and comfortable. Java programmers will probably want to focus on the differences between Java and (C#) to use the (C#) language effectively.

7. **(C#) Versus (C++)**

   While it is possible to program in (.NET) with (C++), it isn't easy or natural. But (C#) is much friendlier.

System Dataflow Diagram

Data Flow Diagram (DFD) or a Bubble chart is a network that describes the flow of data and processes that change, or transform, data throughout the system as shown in figure 43. This network is constructed by using a set of symbols that do not imply a physical Implementation. It is a graphical tool for structured analysis of requirements. DFD models a system by using external entities from which data flows to a process, which transforms the data and creates, output-dataflows which go to other processes or external entities or files. Data in files may also flow to processes as inputs. There are various symbols used in a DFD. Bubbles represent the processes. Named arrows indicate the data flow. External entities are represented by rectangles and are outside the system such as vendors or customers with whom the system interacts. They either supply or consume data. Entities supplying data are known as Sources and those that consume data are called sinks. Data are stored in a data store by a process in the system. Each component in a DFD is labelled with a descriptive name. Process names are further identified with a number. DFD's can be hierarchically organized, which help in partitioning and analysing large systems. As a first step, one Data Flow Diagram can depict an entire System which gives the system overview. It is called Context Diagram of level (0) DFD. The Context Diagram can be further expanded. The successive expansion of a DFD from the context diagram to those giving more details is known as levelling of DFD. Thus, a top-down approach is used, starting with an overview, and then working out the details. A data flow diagram illustrates the processes, data stores, and external entities in a business or other system and the connecting data flows. The four components of a Data Flow Diagram (DFD) are:

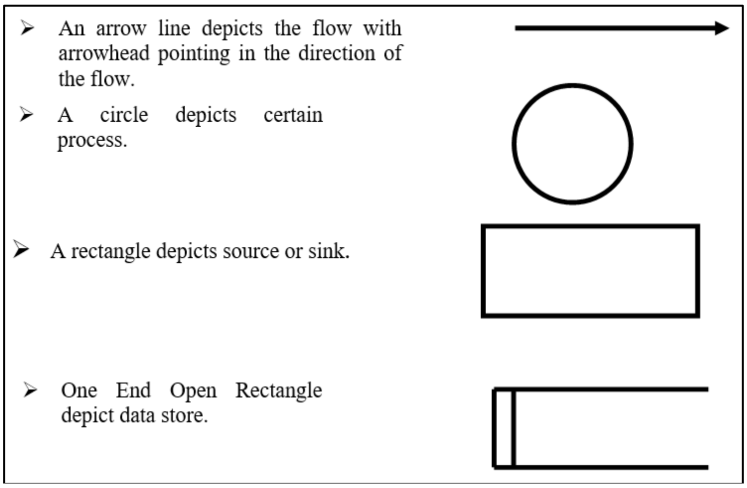


Figure 43: The 4 Components of a Data Flow Diagram (DFD)

Use-Case Diagram

A use-case corresponds to a sequence of transactions, in which each transaction is invoked from outside the system (actor) and engages internal objects to interact with one another and with the system's surroundings. An actor represents a category of user rather than a physical user. Several physical users can play the same role. A use-case diagram is a group of actors, a set of use cases enclosed by system boundary, communication (participation) association between the actors and the use-cases, and generalization among the use-cases.

Sequence Diagram

They are the easy and intuitive way of describing the behavior of a system by viewing the interaction between the system and its environment. A sequence diagram shows an interaction arranged in a time sequence. A sequence diagram has two dimensions: the vertical dimension represents time, and the horizontal dimension represents different objects. The vertical line is called objects lifeline. The lifeline represents the objects' existence during interaction.

Activity Diagram

An activity diagram is a variation or special case of a state machine in which the states are activities representing the performance of operations and the transactions are triggered by the completion of operation. The purpose of the activity diagram is to provide a view of flows and what is going on inside a use-case or among several classes. An activity is shown by round box, containing the name of the operation. The concurrent control is represented by multiple arrows leaving a synchronization bar, which is represented by a short thick bar with incoming and outgoing arrows as shown in figures 44-53.

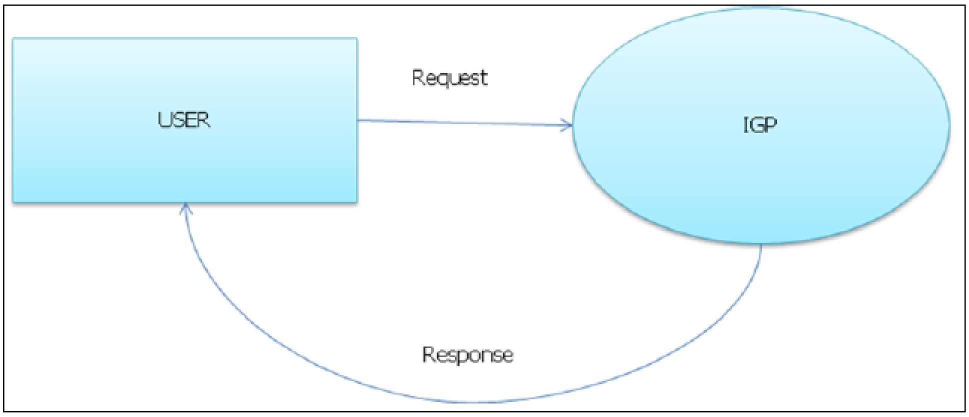


Figure 44: Dataflow Diagram Level (0)

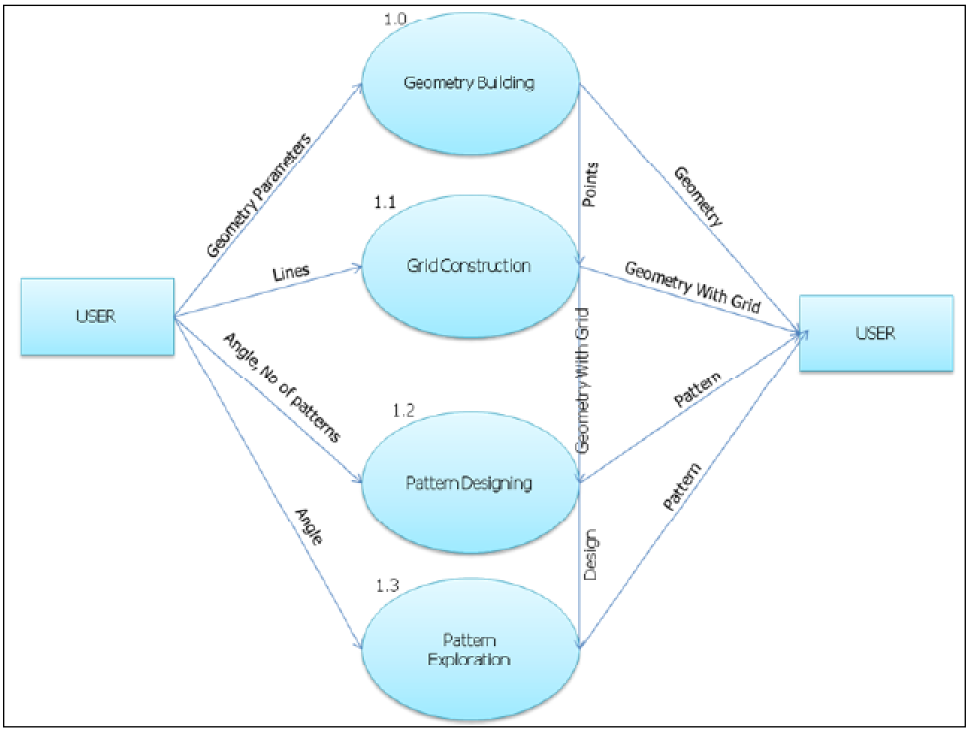


Figure 45: Dataflow Diagram Level (1)

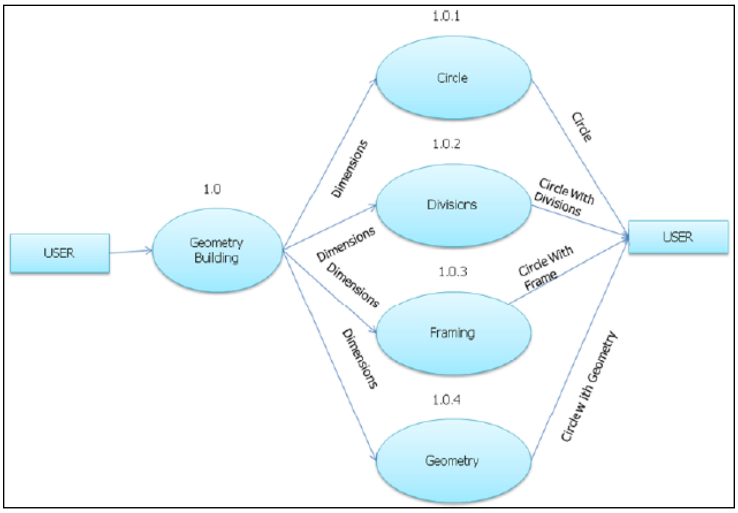


Figure 46: Dataflow Diagram Level (2)

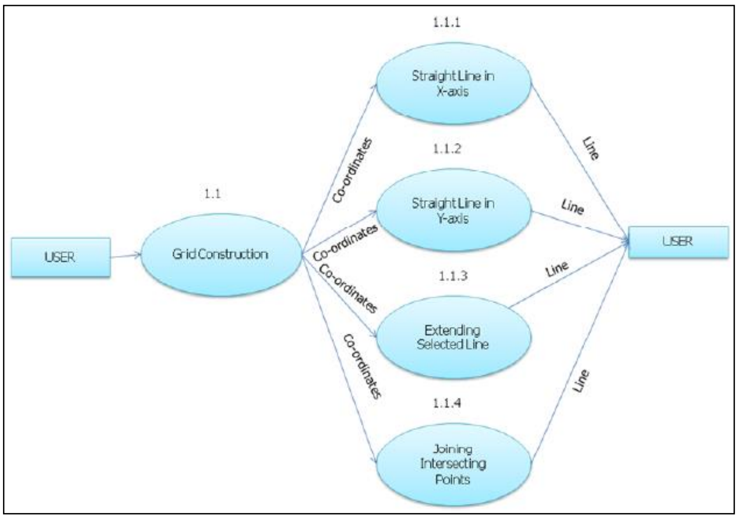


Figure 47: Dataflow Diagram Level (3a)

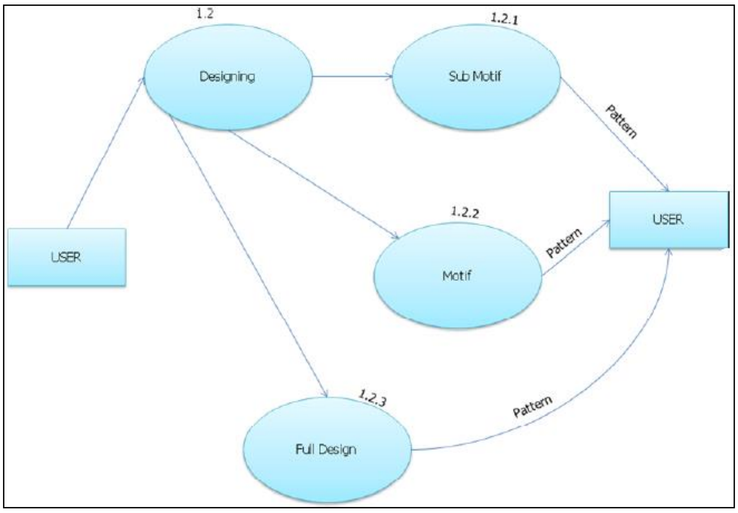


Figure 48: Dataflow Diagram Level (3b)

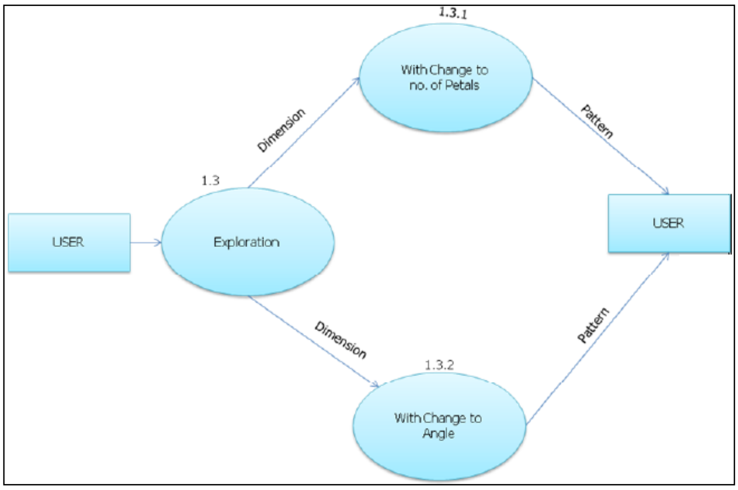


Figure 49: Dataflow Diagram Level (3c)

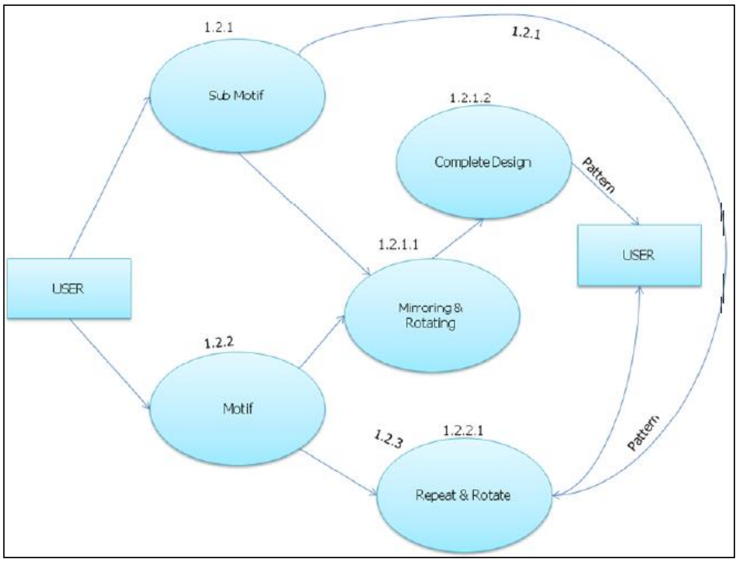


Figure 50: Dataflow Diagram Level (3d)

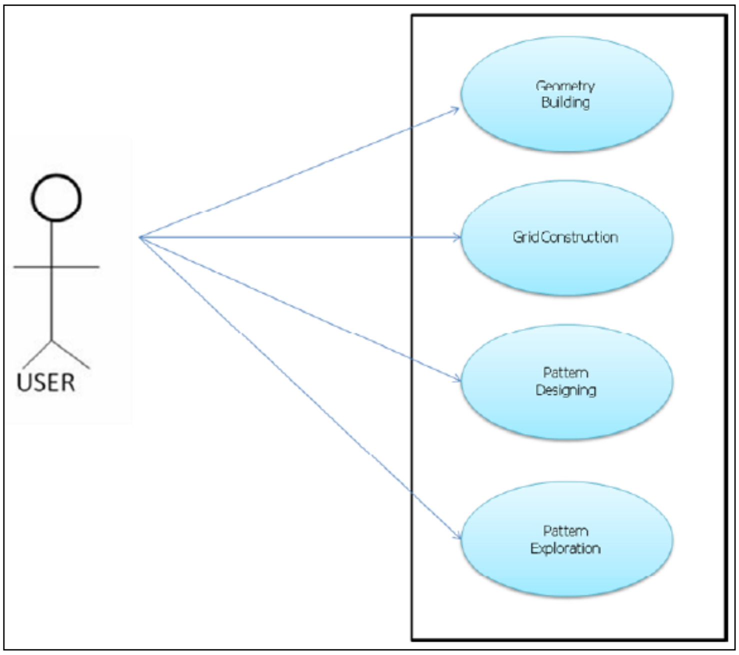


Figure 51: Use-Case Diagram

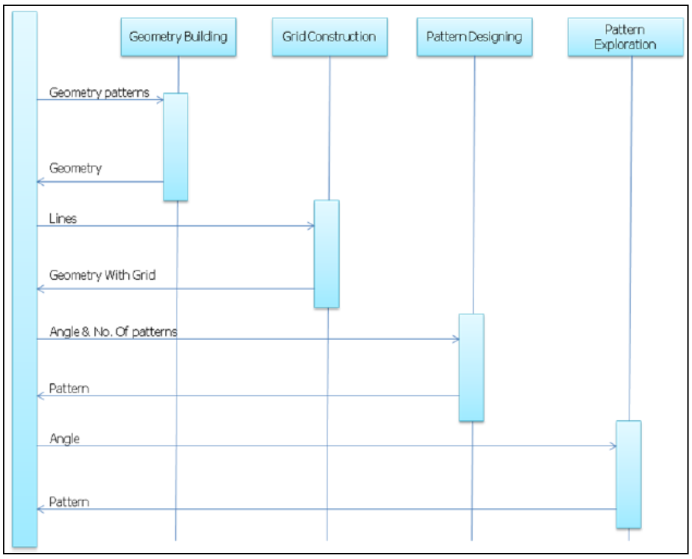


Figure 52: Sequence Diagram

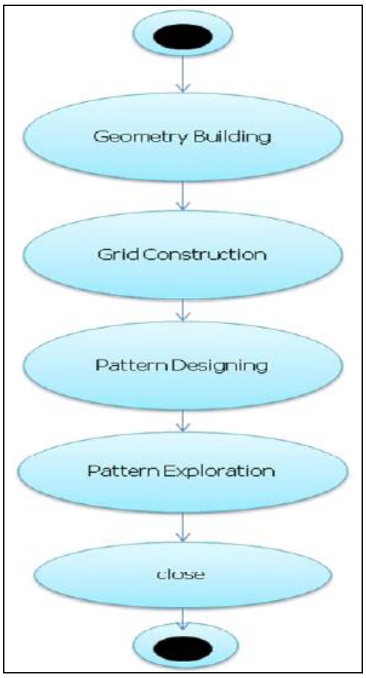


Figure 53: Activity Diagram

System User Interface

This includes designing the visual appearance defining working space, menus, colours, objects, and boundaries. In this process, it is important to develop a Human-Computer Interaction (HCI) that permits naive users to operate easily and provide data about what works as anticipated and what does not work. For conducting this research, the user interface is designed to give the user control over the main functions of the program. The tasks performed by the user are very simple, easy to learn, and do not require any specialized skills or experience. Nevertheless, to be able to properly perform the selection tasks required by the program, the user is assumed to have some basic knowledge about IGP.

System Testing

Testing is the penultimate step of software development. An elaborate testing of data is prepared, and the system is using test data. While doing testing, errors are noted, and corrections are made. The users are trained to operate the developed system. Both hardware and software security are made to run the developed system successfully. System testing is aimed at ensuring that the system works accurately before live operation commences. Testing is very vital to the system. System testing makes a logical assumption that if all parts of the system are correct, the goal will be successfully achieved. The following are the testing methodologies:

Black Box Testing

The technique of testing without having any knowledge of the interior workings of the application is Black Box Testing. The tester is oblivious to the system architecture and does not have access to the source code. Typically, when performing a black box test, a tester will interact with the system's user interface by providing inputs and examining outputs without knowing how and where the inputs are worked upon.

White Box Testing

White box testing is the detailed investigation of internal logic and structure of the code. To perform white box testing on an application, the tester needs to possess knowledge of the internal working of the code. The tester tests inside the source code and finds out which unit/chunk of the code is behaving inappropriately.

Unit Testing

Here they test modules individually and integrate the overall system. Unit testing focuses on verification efforts even in the smallest unit of software design in each module. This is also known as “Module Testing”. The modules of the system are tested separately. This testing is carried out in the programming style itself. In this testing each module is focused to work satisfactorily as regard to expected output from the module. There are some validation checks for the fields. In GP, the unit level testing has been conducted on the software and generated output successfully.

Integrating Testing

Data can be lost across an interface; one module can have an adverse effect on others; sub-functions when combined may not produce the desired major functions; integration testing is a systematic testing for constructing the program structure. While at the same time conducting to uncover errors associated within the interface? The objective is to take unit tested modules and to combine them and test it.

User Acceptance Testing

User acceptance of a system is the key factor for the success of any system. The system under consideration is tested for user acceptance by constantly keeping in touch with the prospective system users at the time of development and making changes whenever required. This is done regarding the following point:

Input Screen Testing

An acceptance test has the objective of selling the user on the validity and reliability of the system. It verifies that the system’s procedures operate to system specifications and that the integrity of important data is maintained. Performance of an acceptance test is the user’s show. User motivation is very important for the successful performance of the system. After that a comprehensive test report is prepared. This report shows the system’s tolerance, performance range, error rate and accuracy. In GP, the input screen testing has been conducted on the software and generated output successfully.

Output Screen Testing

After performing the validation, the next step is output testing of the proposed system, since no system could be useful if it does not produce the required output in the specified format. Getting necessary information from the user about the format required by them and tests the output generated or displayed by the system. In GP, the output testing has been conducted on the software and generated output successfully.

Chapter 4: Implementation and Experimental Results

The goal of this phase is to implement the software correctly, efficiently, and quickly for our task. Coding is the first stage of implementation activity. This is usually the longest phase involved in writing algorithms. Testing is the second stage which involves testing using sample data, debugging the software, and testing information processing procedures; revising the algorithms and correcting faults is the third stage. Finally, the last stage is evaluating the results. These four tasks must work simultaneously to reach the final design.

Input Operational Guideline

Input design is one of the most expensive phases of the operation of computerized system and is often the major problem of a system. A larger number of problems with a system can usually be traced back to fault input design and method. Therefore, that the input data is the life block of a system and must be analysed and designed with utmost consideration. The decisions made during the input design are:

1. To provide cost effective methods of input.
2. To achieve the highest possible level of accuracy.
3. To ensure that input is understood by the user.

System analysts decide the following input design details like, what data item to input, what medium to use, how the data should be arranged or coded which are data items and transaction needing validations to detect errors and at last the dialogue to guide users in providing input. The IGP Input parameters are as follows:

1. Dimensions for Circle formation.
2. Dimensions for Frame.
3. Selection of Geometry Type and Dimensions for geometry drawing as well as number of geometries.
4. Dimensions for Grid like pen size, color, co-ordinates for drawing intersecting lines, straight lines, extending lines etc.
5. Dimensions for sub-motif / motif, selection of mirror & rotate, repeat & rotate, co-ordinates for drawing intersecting lines for design, and completion of design drawing permission.
6. Provide several patterns to build for tiling.
7. Provide exploration values of Petal Numbers.

Output Operational Guideline

Output design generally refers to the results and information that are generated by the system. For many end-users, output is the main reason for developing the system and the basis on which they evaluate the usefulness of application. The objective of a system is to find its shape in terms of the output. The analysis of the objective of a system leads to determination of outputs. Outputs of a system can take various forms. The most common are reports, screen displays, printed form, graphical drawing etc. The output also varies in terms of their contents, frequency, timing, and format. The users of the output, its purpose and sequence of details to be printed are all considered. The output from a system is the justification for its existence. If the outputs are inadequate in any way, the system itself is inadequate. Output design phase of the system is concerned with the convergence of information to the end user-friendly manner. The output design should be efficient and intelligible so that the system relationship with the end user is improved and they will enhance the process of decision-making. The basic requirements of output are that it should be accurate, timely and appropriate, in terms of content, medium and layout for its intended purpose. Hence it is necessary to design output so that the objectives of the system are accomplished in the best possible manner. The outputs are in the form of reports.

Classification and Design Software Structure and Description

The software structure is an extremely detailed set of operations and sequence of instructions written to perform the task of classification and design. The software has an executable form that the computer can use directly to execute the instructions. The same software in its human-readable source code form, from which executable software are derived, enables a programmer to study and develop its algorithms. software structure and organization represent the planning phase of the whole process. Detailed description about the output designs for each input are given below:

01 On launching the software, the interface displays a black screen with several menus on top left showing file menu, edit menu and classification menu.

02 After giving values to the radius, unit of measurement, angle of division, pen size and color options in Classification menu, the program inputs a circle based on given values.

03 The software classifies the circle with its angle parameter and displays on the screen the 1st part of our naming convention for classification.

04 Based on user selection in the classification menu for Frame and its pen size and color, the software inputs a frame around the circle with selection of pen size and color.

05 Also, in the classification menu user can input details of the geometry: triangular (triangle, hexagon and nonagon), quadrilateral (square and octagon) pentagon and heptagon with its pen size, color and number of geometries required.

06 The program reads the geometry parameters and displays the selected geometries inside the circle on the screen. The geometry will be automatically placed at equidistant angle from each other inside circle.

07 The software classifies geometry and displays on the screen the 2nd part of our naming convention for classification.

08 Based on user's selection, the program proceeds to the second phase of classification menu by clicking the Grid option in the classification menu.

09 On clicking the Grid Menu, the interface shows several more options to draw more grids.

10 START GRID button: user can draw unlimited grid lines over the circle and geometry.

11 Options for drawing grid lines are given to extend a line, draw join line between intersecting points on any line already drawn, draw line through x-axis or through y-axis etc.

12 Whenever user draws a new line on the drawing area, if that line intersects any previous lines or circle, all such points will be shown as active drawing points. New lines can be drawn connecting these intersecting points.

13 After finishing the grid lines drawing, the user needs to click the END GRID button.

14 Application will register the drawn grid on the left side of the screen with numbering.

15 Users can create as many grids as possible over the circle through the same procedure. All these grids will be temporarily saved on the left side of screen. Thus, the user can view the number of grids drawn for final classification and design.

16 BACK button: user can go one step backward.

17 DELETE button: user can delete the selected grids from the grid list registered on the left side of the interface.

18 VIEW GRID button: user can view all grids at the same time.

19 STRIGHT LINE button user can draw straight lines in x-axis or y-axis from the point of clicking to the border of the frame with selected pen size and color.

20 EXTENDED LINE button: user can extend the geometry lines to the border of the frame with given pen size and color.

21 CLASSIFY button: user is entering to end the classifying stage.

22 The program reads the CLASSIFY button selection and classifies the IGP grid and displays the final classification on the screen.

23 By selecting the SUB-MOTIF button user can select the smallest portion of the drawing area including circle, frame, division on circles and grids.

24 The program reads the SUB-MOTIF selection and automatically generates the SUB-MOTIF portion grid on screen. i.e., according to the angle of smallest portion given for SUB-MOTIF, program keeps only given angle's area up to the frame and removes rest of the portion from the drawing area.

25 Also, users can select from the SUB-MOTIF menu different parameters such as repeat & rotate, mirroring & rotate, angle, pen size and color to be used for design and exploration of GP IGP.

26 MOTIF button: user can get a double sub-motif portion of the grid in design.

27 The program reads the MOTIF selection and automatically generates the MOTIF portion grid on screen.

28 Users can also select from the MOTIF menu different parameters such as repeat & rotate, mirroring & rotate, angle, pen size and color to be used for design and exploration of IGP.

29 FULL DESIGN button: user gets the full-size IGP design.

30 The program reads the FULL DESIGN selection and automatically generates the full design and displays on screen.

31 TILE button: user can generate an GP IGP pattern.

32 User can change the x-y direction parameters for the n-number of units in x-y direction.

33 The program reads the TILE selection and automatically generates the GP IGP pattern and displays on screen. By selecting the PETALS button from the exploration menu user can change parameters of the design.

34 The program reads the new parameters of the PETALS and automatically generates and displays the explored GP IGP design by selecting the ANGLE button from the exploration menu user.

35 The program reads the new parameters of the ANGLE and automatically generates and displays the explored IGP design.

36 COLOR button: from the fill color menu user can change parameters of the color inside the close boundaries of the design.

37 The program reads the new parameters of the COLOR and automatically generates and displays the IGP design on screen.

Experimental results

Usually, IGP designs are named and classified on basis of their visual appearance such as pentagonal, hexagonal, heptagonal, octagonal, nonagonal, etc... Such classification is too general and misleading. Therefore, instead of looking at IGP designs in terms of visual appearance, it is proposed to classify them based on their gird system attributes according to normalization grid methodology, Normalization, which is a process of reducing to a norm or standard, can be done by identifying individual geometries and grids that make up the unit designs of IGP. using proposed software could be taken up in the following stages as are shown in figures 54-66.

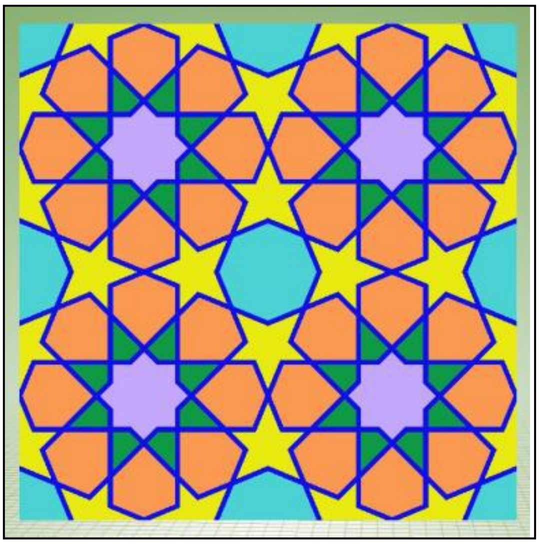


Figure 54: Identifying the Pattern

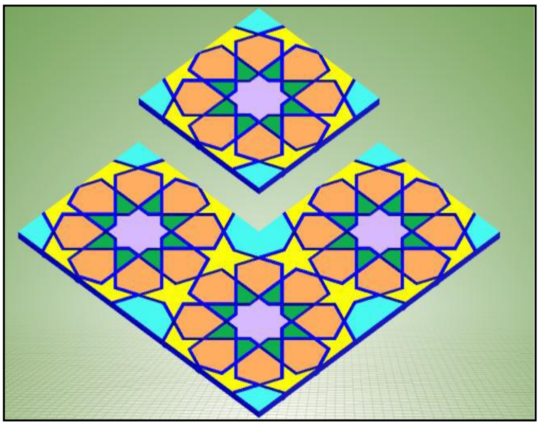


Figure 55: Identifying the Unit Pattern

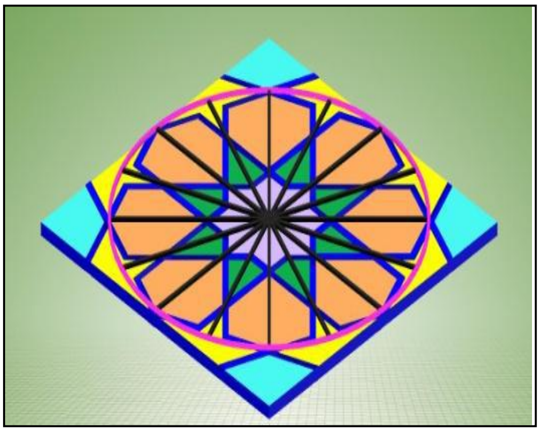


Figure 56: Identifying Division of the Circle

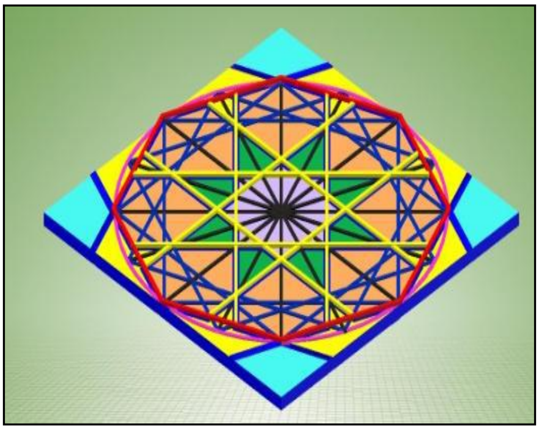


Figure 57: Identifying the Grid System

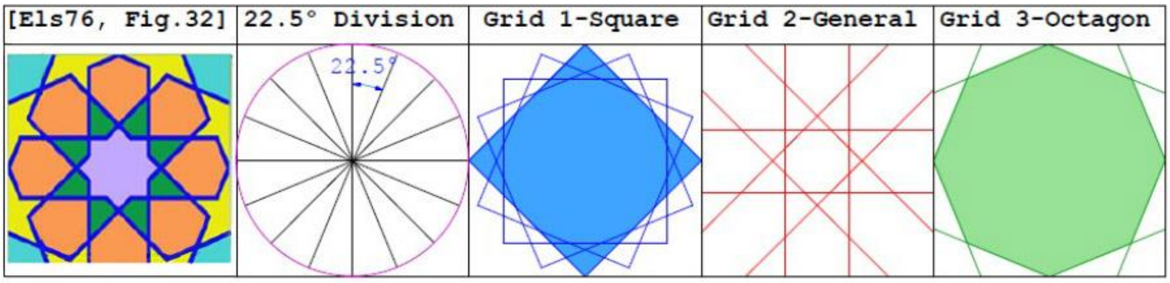


Figure 58: Final Classification 22.5° Triple Grid Quadrilateral (22.5° 3Q)

The Classification Stage - Circle Division

In this classification stage, the parameters of the circle are initiated by choosing the radius of the circle (R) which restricts the placement of grids within the parameters of the size. In the same menu the circle can be divided into (x) number of cones based on the design, and the user can select the unit of measurement, pen size and pen color. In this example the circle parameters are set for; circle color red, pen size 2, circle size 3cm, angle 22.5°, divides the circle into 16 segments. Automatically the software displays the classification angle on the screen. The display of the angle on the screen is the 1st part of our naming convention as shown in figure 59.

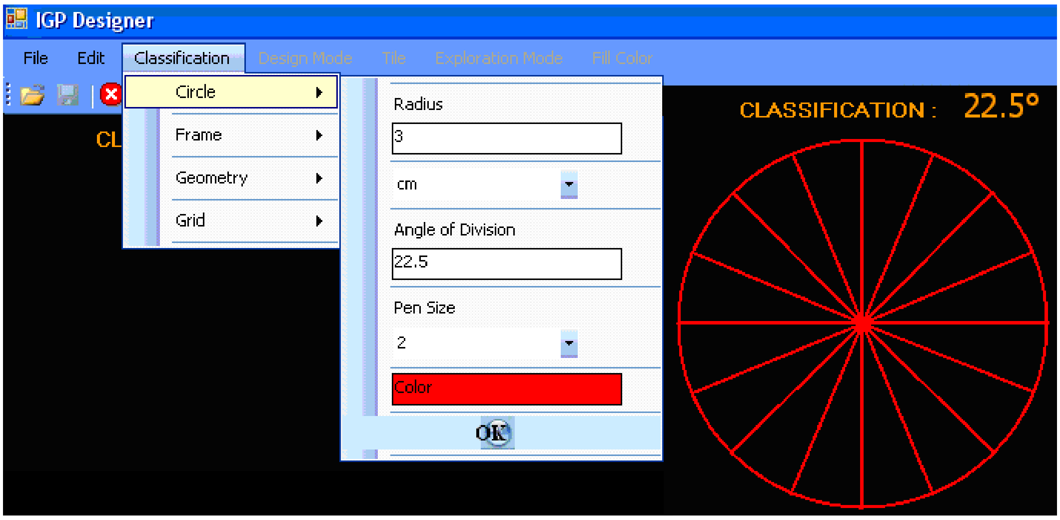


Figure 59: IGP Software Generating the Circle with 16-line Divisions at 22.5° Angle in Red Color

The Classification Stage - Geometry Selection

At this classification stage, the user can choose the numbers and types of geometry to design the grid system of an IGP design (triangular, quadrilateral, pentagonal or heptagonal). In our example, we are selecting 4 squares from the quadrilateral menu inscribed in the circle drawn at equidistant points, also, we are selecting an octagon. Automatically the classification software displays the type of geometry selected. The display of the geometry on the screen is the 2nd part of our naming classification convention as shown in figures 60-61.

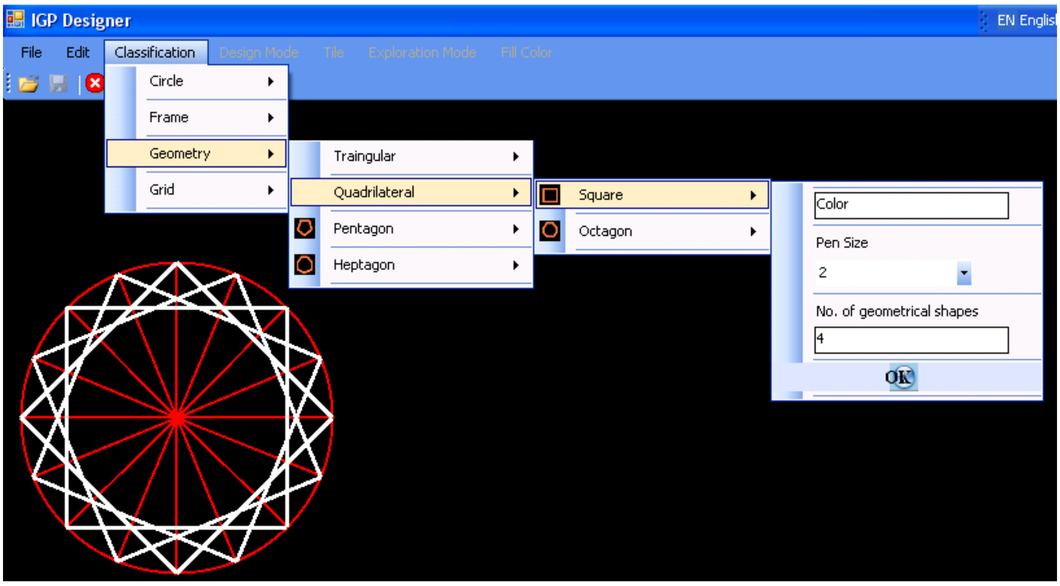


Figure 60: IGP Software Generates 4 Square Grids in White Color

Figure 61: IGP Software Generates Octagonal Grid in Yellow Color

The Classification Stage - Frame Selection

At this classification stage, user can circumscribe the circle with a frame (external boundary). The external boundaries are usually a square or a rectangle, are regarded as indispensable in some designs. The external boundaries are associated with extended lines of grids and geometries in the design. The external boundaries are phantom to the IGP design, and its presence cannot be always confirmed until and unless the existence of external mesh can be traced as shown in figure 62.

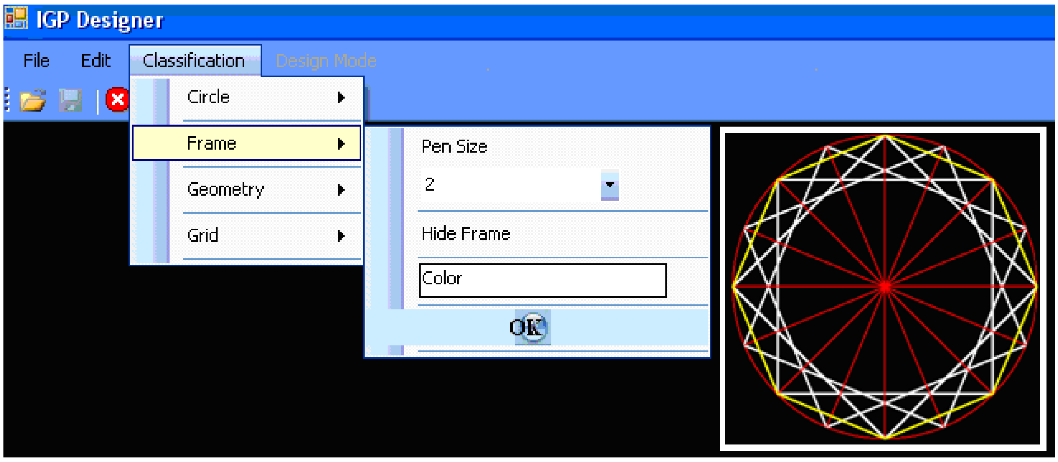


Figure 62: IGP Software Generates the Boundary Square in White Color

The Classification Stage - Extension and General Grid Selection

At this classification stage, user can build array of general grids. After finishing each general grid user must click Finish for the software to store and count total grids. When the grid stage is finalized, by clicking the button Classify the software will classify the design. The displacement of the number of grids on the screen is the 3rd part of our naming classification convention. In some IGP designs the grid lines needs to be extended to accommodate the design. The extension feature has been added to the software where the required grid lines extended to border the notational boundary (the frame). The extended lines of grids and geometries would generate intersection points to accomplish the seamless mesh in the external zone within the notational boundary as shown in figure 63.

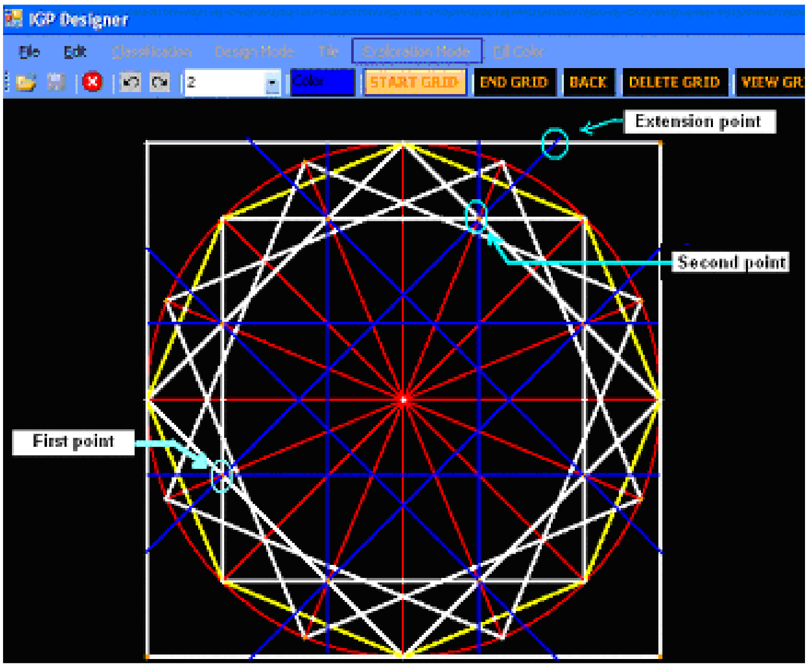


Figure 63: IGP Designer Software Capable of Extending the Grid Lines for Final Classification

The Classification Stage – Final Classification

At this stage the classification process is complete and displayed on the screen (22.5° 3Q) where the grid system of the IGP is classified as "22.5° triple grid quadrilateral" as shown in figure 64.

1. (22.5°) represents the circle division and the angle of the sub-motif.
2. (3) represents the number of grids used.
3. (Q) denotes the quadrilateral class type.

In the classification naming convention software logic has been adopted to normalize types of geometries used into their norms. In the above example, the design requires 4 squares and 1 octagon. When the Octagon is normalized it makes 2 squares. Therefore, the software classifies the naming convention as quadrilateral. Similarly, when triangles or their multiples of geometry are used, the software logic will normalize the geometry into triangular classification.

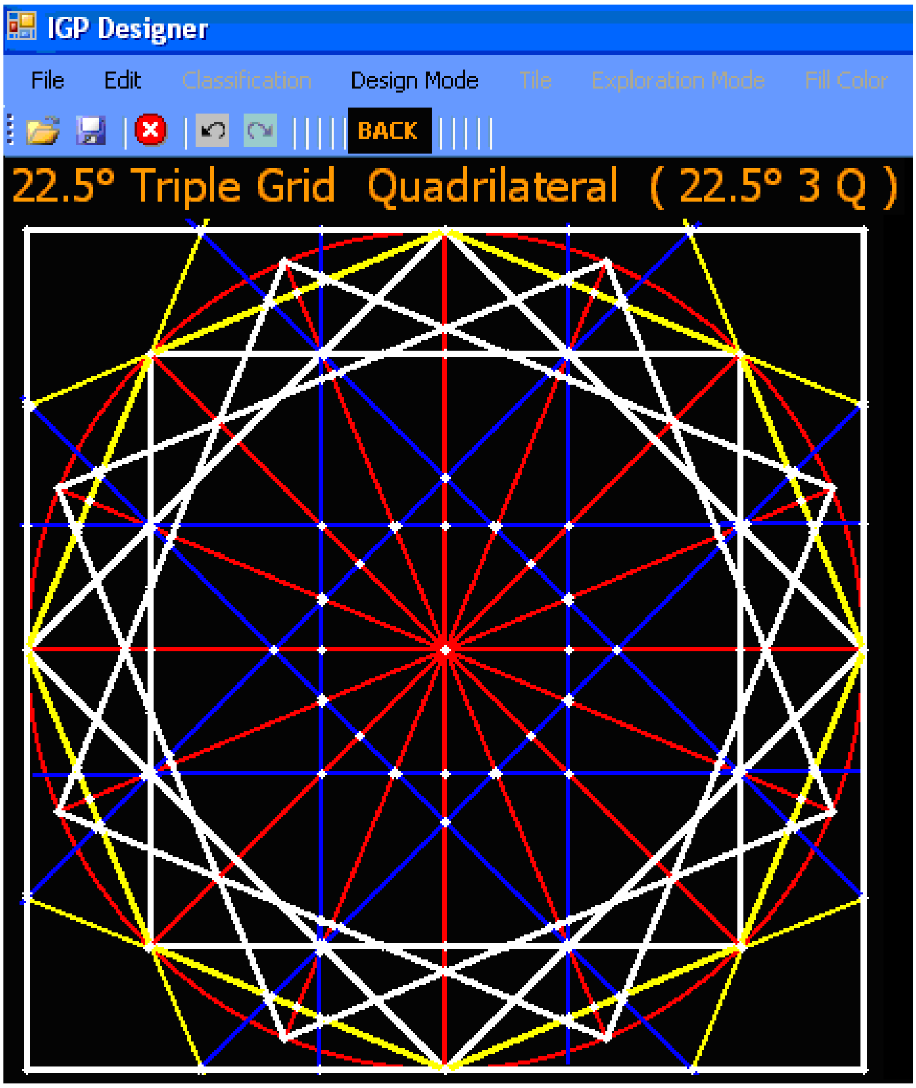


Figure 64: IGP Software Classifies the IGP as (22.5° 3Q) Using 3 Grids Quadrilateral Type

The Design Stage - Motif Selection Stage

The motif selection stage initiates the designing process and finally the pattern development. The core objective of this stage is to select the sub-motif. And this is done automatically by the software recognizing the angle of division of the circle as the angle of the design and cuts the design grid view into the Sub-Motif. The angle can be altered to Motif or even Full-Patter as per the user. In the Sub-Motif designing stage the user has the choice of either Repeat & Rotate or Mirroring & Rotate and other parameters. Finally, the user can develop the Full Pattern in x-y direction as shown in figure 65-66.

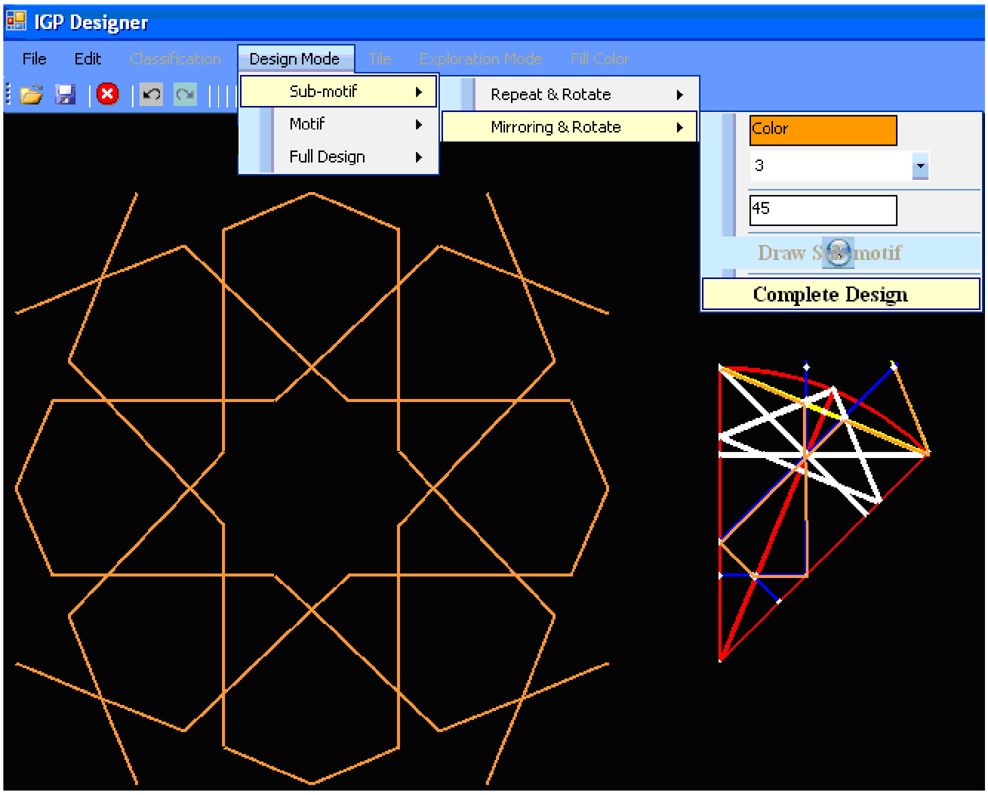


Figure 65: IGP Software Generates the Sub-Motive to Draw the Unit Pattern by Mirroring and Rotating

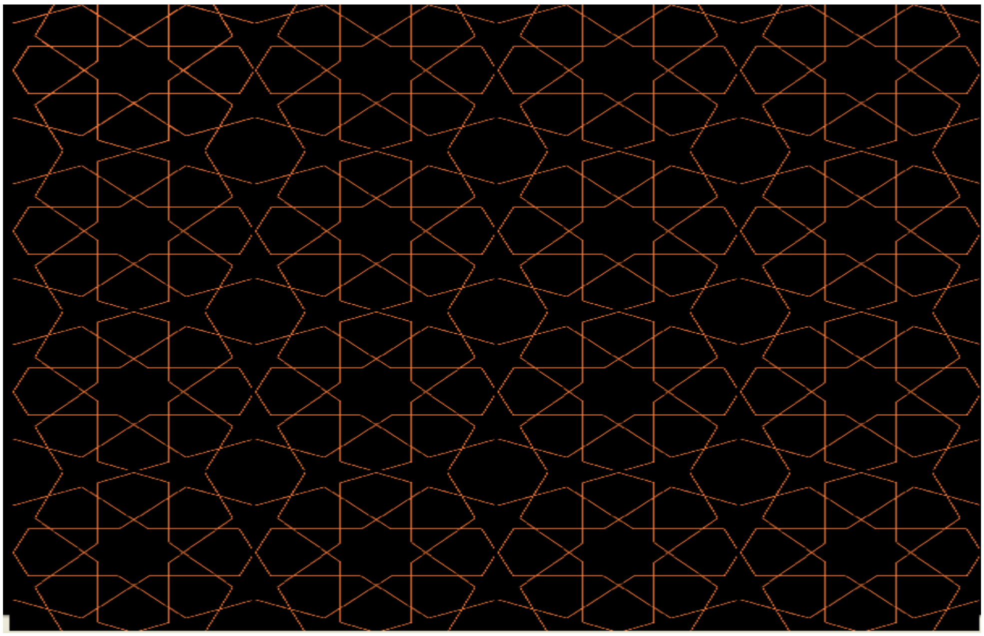


Figure 66: IGP Software Generates the Pattern in x and y direction

Chapter 5: Conclusion and Future Works

In conclusion, the approach of this thesis is unique as each chapter complements the following, sacred meanings, design methodology, classification, and software design, which makes their correlation exciting research.

Sacred Meaning Conclusion

In the sacred meanings task, to refute the conclusions of some scholars who said there are no sacred meanings behind IGP designs. Nearly a millennium ago, artists had concrete thoughts lingering on this subject well documented by ancient books. Solid evidence presented under the aegis of the works of ancient Islamic scholars [Ala1823] [Alb1225] discussed in this thesis. It was essential to have a consistent Islamic backdrop and never ventured into any modern work by authors of the current age and showed the inspiration behind the evolution of the IGP grid system and design. When the designer gives us a holistic view of a given design from practical and spiritual dimensions, we can fully understand the design. If the artist does not base his IGP design under the benchmarks of sacred meanings, it will not be able to fully justify its subject. There certainly has been a spiritual benchmark behind the accomplishments of the IGP designs over the ages.

Another dimension was established in the sacred meanings of IGP, revealing various studies; there have been illustrations of designs without studying the associated sacred alphabets and numerals. In all future works, the researcher would like to decipher the meanings of sacred alphabets and numerals associated with designs. Such a domain would interest artists, architects, interior designers, and historians.

Classification Conclusion

In the classification task, we believe it's inappropriate to classify IGP based on 7-frieze and 17-wallpaper group theories simply because they are based on mathematical transformation, and IGP is symmetrical in nature; therefore, any mathematical transformation based on 7-frieze and 17-wallpaper group theories does not affect the IGP design and arrangement as has been illustrated in this thesis.

This thesis stated the problem with a clear objective of classifying IGP based on their grid system and geometric attributes. The classification gives the reader information on the design angle, geometry type and grids used to achieve the design.

The theorem presented in this research enables us to classify IGP based on their grid system.
For IGP grid classification, we based our technique on the grid normalization methodology meaning the triangle, quadrilateral, pentagon, heptagon, and nonagon are the basic geometries, and other geometries are just the multiples of that; for example, hexagon is the multiple of triangle and octagon is the multiple of quadrilateral, etc. It also generates a conventional classification naming based on grid normalization methodology, which gives the reader information on the minimum grid system used to generate IGP designs. The results obtained by grid classification can be relative according to artistic experience. There is always room for a new critic to classify the same IGP design using different parameters and methodologies.

Design Methodology Conclusion

On the design part, we found that it has led to several significant design methodologies that advocate the grid system, designing, and exploration in different ways. Design methodologies are still evolving, and this field may take some time to boast. Each design methodology has its unique proposition, norms, and way of constructing IGP. The grid system has an inherent benchmark for measuring the mechanisms used to design IGP. When we explore the designs under the unconventional light of the grid system, we can see varied dimensions of the art form, which would facilitate their construction and deconstruction. The ruler-compass method [Els93] is essential to gain a fundamental understanding of the hierarchical formation of the designs. Within El-Said's work, there is no clear emphasis on the motif. Salman [Sal91] has done great work on related areas, and they are numerical and presented in fixed parameters.

On reflection of the different design methodologies, the grid normalization methodology does utilize the symmetry of the form and the symmetry of the underlined basic shapes. Grid normalization methodology gives the artists freedom of work with no restrictions on the grid system by plugging the radius, types, and number of geometries.

However, the grid normalization methodology's key advantage is that it allows us to explore new designs. The grid normalization approach toward design is more flexible and productive since it produces the original design and demonstrates varying forms of changing input parameters for the output of classification and new designs. The fundamental principles of all these design methodologies vary based on the experiences and disposition of their proponents.

Software Design Conclusion

The general goal of this research was to develop software that can classify and design IGP. Visualization techniques are practical tools that can substitute human perception in performing some tasks. Visualization techniques are more capable of recognizing complex IGP designs than traditional methodologies. Although the researcher focused on the grid system of Issam El-Said IGP design, the underlying goal was to develop general ideas and principles that might be applied to other IGP styles. We have developed a new design methodology in conjunction with classification using the grid system, which can be used to design based on the motif grid. Since all IGP designs share similar general properties, applying the same conceptual method to construct, classify and design is possible. Its hypothesis is that the software offers many advantages over traditional methodologies. The findings have proven the validity of the research and provided ample proof of the concept proposed in the hypothesis. This thesis has aimed to explore the possibility of using modern mathematics and computer sciences to develop new ways to classify and design IGP.

The researcher developed an ambitious software using C Sharp on the .NET platform. We developed software utilizing algorithmic mathematics to analyze, construct, classify, motif detection, design and explore IGP. The proposed software has proven successful in constructing accurate IGP designs on the analysis of ruler-compos methodology to understand the hierarchical formation of IGP designs and in categorizing the exact motif of more complex grid systems. The software was developed based on grid normalization methodology and grid system to automate the IGP classification and design. However, there could be a few glitches in the program's source code, or specific runtime errors could occur unexpectedly during runtime.

Based on research findings, here are lists of summaries drawn from the research: the software was challenged to explore new designs. The grid normalization methodology applied to generate IGP grid classification and design indicates the success of the software developed. Nevertheless, it can also explore new IGP designs by drawing over the motif using input parameters, angles, and petals. The software proved efficient in capturing the correct motif based on the grid system. The software enables users to create IGP from scratch in stages and quickly arrive at their classification. Also, demonstrated an approach to discovering the infinite possibilities of new designs beyond ever done before based on mathematical parameters.

The effort of this research is not directed toward developing professional software; instead, it only provides a solution to the idealized problem. Nevertheless, it represents the first step in that direction. This research hopes to draw more attention to the importance of utilizing new technologies by incorporating some built-in intelligence into visualization technologies and enabling the creation of powerful tools that can perform classification and design easily, quickly, and more efficiently than traditional methodologies. The successful outcome of this thesis would not only offer a rich resource for art communities, but also it would be of interest to mathematicians, crystallographers, architects, archaeologists, and others to explore IGP in their artwork. Future research should define the margin of error that occurs when using the computer software and define areas of defects in the code, which can be modified to get better results. It's an excellent tool for designers, architects, mathematicians, and academic fraternities working in related areas.

The software does possess a successful dry run as far as the end-to-end functionality is concerned. But certain sections of the application where menu options are present are still being written and tested. The researcher is committed to perfecting software, giving it a firm and final form and shape, and enhancing it to give it a robust performance platform. For the development of algorithms, processing strategies and compression, which developed the software, different types of design can be produced with different input parameters to output logical classification and design. It also has an excellent capacity for testing many quick, accurate, and easy designs compared to the limitations of human power. This research is concerned with the qualitative testing of design accuracy and repeatability. They can serve as elegant test beds for programming and texture mapping research. The software is aimed at holding the data in a relational database management system that can index and categories designs based on their set attributes. This would enable us to store and catalogue many designs to make them available in an online library for scientists, mathematicians, designers, geometricians, architects, interior decorators, and academic researchers.

Contributions

This thesis grew out of the use of computer graphics and program tools to classify and design a geometric pattern IGP. As such, the goals were discovered along the way as researcher's ideas and techniques developed and became more powerful. In general, the researcher cannot aim to achieve a specific goal or inspire a specific visual response, but when some interesting result is found, we can then reflect on the method that produced that result and its applicability to other problems. Here are the main contributions that this work makes to the greater world of computer graphics and program tools.

1. **A Model for Classification**
   One specific contribution is the development of a model describing a sophisticated theory that can account for the classification of a wide range of geometric pattern IGPs. This theory is used to create a novel classification based on a geometric pattern normalisation methodology and grid system.

2. **A Model for Design and Exploration**:
   Other main contribution is the development of a model describing a sophisticated theory that can account for the design and exploration of a geometric pattern IGPs. The model accounts for design and exploration based on a geometric pattern design angle and motif. The generated algorithm can help a geometric pattern designer to create novel geometric pattern IGPs from scratch.

3. **CAD/CAM**
   The researcher's attempt is to develop an ambitious software (a geometric pattern Designer) developed on (C#) in (.NET) platform to analyse, classify, design and explore a geometric pattern IGP. Computer modelling of a geometric pattern IGPs provide a new dimension in their algorithmic representation for further exploration and their application in the modem artistic design. This is likely to provide a new framework for its visual expression that extracts a set of new transformation of forms while the underline messages of symbolism remain intact.

4. **Other Work**
   This section presents some other works that are generally related to the computer generation of GP IGPs.

4.1 Floral

In floral design a pioneering work has been presented by Wong et al. Their work provides a modern approach in the analysis and generation of floral design, as well as arrangement and classification. The algorithm that generates the floral design decomposes a collection of primitive motifs and once their layout is finalized, the proxies are then colonized over a given region. The approach leaves the creation of the suitable motifs to the artist. The work of Wong et al. is valued as having a significant contribution and considered as innovation by using computer graphics and preserving the tradition [138]. Kaplan points out that Wong et al. approach is very loose and not constrained by global order such as symmetry although the approach lists repetition as a principle of design. He also points out that the approach is designed for small surfaces and might be less successful in large architectural projects [31].

4.2 Fractal

One of the complex designs that are associated by computers as a powerful tool for creating the fractal design which has a high degree of order, but little symmetry using the Mandelbrot set that need precise computation and high order repetition. Many computer scientists continue to research interesting ways to render the Mandelbrot set and fractals. Chaos theory is closely related to fractal geometry. Field and Golubitsky have created many fractal designs by using dynamical systems which have wallpaper symmetry [139]. Many scientists and artists find a rebirth of decoration design in the digital age [31].

4.3 Celtic Knotwork

Celtic knots are stylized graphical representations of knots used for decoration, adopted by the ancient Celts. The interlace patterns are found in Byzantine architecture, late Roman Empire Coptic art, Celtic art, Islamic art, Ethiopian art, and European architecture. However, there is no evidence to indicate that a knot had any specific philosophical or religious significance. At some point after the 9th century the techniques used to create Celtic art were lost. Research into both Celtic knotwork and Islamic star patterns has required the unravelling of historical mysteries. J. Romilly Allen has identified eight elementary knots which form the basis of nearly all the interlaced in Celtic designs [140]. George Bain recreated many Celtic designs based on breaking crossings [141]. His son, Iain Bain simplified his father's methods to be algorithmic based on a tri-grid system [142]. Meehan has published a series of books extending Iain Bain's work [143] [144] [145].

Peter R. Cromwell presents a construction method like Bain's, based on an arrangement of two dual rectangular grids [146]. Wong et al. pointed out there has been little work in computer generated ornamentation [138]. Matthew Kaplan and Elaine Cohen presented a technique for automating the construction of Celtic knotwork. They also show how to use such knotwork in 3D and demonstrate a variety of applications including artwork and transforming the designs into 3D models for fabrication [147]. At present, Celtic artwork popularity has manifested itself in design, fine arts, jewellery, body art, decoration of sculpture, and architecture [31].

Future works

Future research should be directed towards examining visualization techniques; shade and shadow lighting processing IGP designs. The semi-automated software developed by the research provides the first step into developing a fully automated process. Full automation is possible based on the results of this research.

There is immense potential and further scope for future work in classification and design. The compilation of a comprehensive database for a set distinct IGP design concerning their sacred meanings, classification, and design will be indexed in many significant categories. To enable artists to create IGP designs from scratch in stages and arrive at their classification and design with ease by using automated software developed using C# on the .NET platform. The proposed normalization grid methodology can also be modified to work with other types of IGP designs that have special characteristics, such as:

1. IGP designs that have curved lines: we need to modify the way to link the defined grid system intersection points to draw the curved lines. This can be done by using available programming techniques.

2. IGP designs covering three-Dimensional surfaces (domes, niches, etc.): we need to incorporate the idea of 3D projection techniques.

3. IGP designs rendered as color areas: we need to apply different approaches for segmentation. Region-based segmentation can be used first and then mapping the boundaries of different regions as the base for extracting line information.

4. Creating Eigenvalues: studying patterns of IGP and creating Eigenvalues for those patterns and saving.Using artificial neural networking: in artificial intelligence, a user when given a new image pattern, from the Eigenvalues comparative study using ANN can be done, so that it will be automatically classified from its just .jpeg format, according to the classification style represented in our current IGP work.

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