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Mr: MOHAMED HLIMI

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Nom Prénom	Titre	Etablissement	
Mahjoub BENGHOULAM	PES	Faculté des Sciences de Meknès	Président
Said SAADEDDINE	PES	Faculté des Sciences et Techniques de Mohammedia	Rapporteur
Kamar OUAZZANI	PES	Ecole Supérieure de Technologie de Fès	Rapporteur
Rachid SAADANI	PH	Ecole Supérieure de Technologie de Meknès	Rapporteur
Tarik KOUSKSOU	PES	Université de Pau et des pays de l'Adour	Examineur
Youssef MOURAD	PES	Ecole Supérieure de Technologie de Fès	Directeurs de thèse
Abdelmajid JAMIL	PH	Ecole Supérieure de Technologie de Fès	

Mustapha MAHDAOUI	Faculté des Sciences et Techniques de Tanger	Invité
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Laboratoire d'accueil : Laboratoire Productique Energétiques et Développement Durable.

Etablissement : Ecole Supérieure de Technologie de Fès.

Dédicace

À MES CHÈRES PARENTS

pour vos sacrifices que vous avez consentis pour mon instruction.

A Mes fils, ma femme, ma sœur, mes frères et toute ma famille

*Pour votre aide précieux et votre patience durant l'élaboration de ce
travail*

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Résumé :

Le domaine du stockage de l'énergie thermique a récemment suscité un intérêt grandissant, vu son application dans différents domaines, tels que le chauffage des espaces intérieurs des locaux, l'exploitation des déchets thermiques, le chauffage de l'eau, le refroidissement et la climatisation... L'utilisation d'un système de stockage par chaleur latente (LHS) utilisant des matériaux de changement de phase (PCM) est un moyen efficace de stocker l'énergie thermique et présente les avantages d'une densité de stockage élevée, en plus de la nature isotherme du processus de stockage. Il existe un grand nombre de PCM qui changent leurs phases à une large gamme de températures, ce qui les rend attrayants dans un grand nombre d'applications. La densité de stockage d'énergie plus importante réduit considérablement le volume de stockage et la température de stockage relativement constante évitent les fluctuations pendant les opérations de stockage et de récupération; or le principal problème quant à l'utilisation de ces matériaux, et qui suscite beaucoup d'intérêt, est que la plupart de ces matériaux ont une conductivité thermique faible, et par conséquent, des techniques d'amélioration du transfert de chaleur sont nécessaires pour toute application de stockage thermique par chaleur latente.

A cause de la complexité des équations mathématiques, due essentiellement à la non linéarité du terme convectif dans l'équation de Navier - Stokes, en plus du terme de diffusion visqueuse lié au transfert de quantité de mouvement, et qui introduit des dérivées de second ordre; la résolution rigoureuse des problèmes, impliquant le changement de phase couplé à la convection naturelle, n'est possible que pour un nombre de cas simples, ce qui rend la modélisation de ces phénomènes d'une utilité capitale.

Dans cette thèse, on s'intéresse à la modélisation du changement de phase solide liquide en présence de la convection naturelle dans différentes encapsulations; le modèle physique utilisé est basé sur la formulation enthalpique, élaborée pour mettre en évidence le couplage entre la convection naturelle et le processus de fusion du MCP. Le schéma numérique utilisé se base sur la méthode des volumes finis, Par ailleurs, l'étude met en exergue l'effet des paramètres géométriques des capsules sur le processus de fusion du MCP dans différentes encapsulations. Durant ces investigations, les formes géométriques qui ont été modélisées sont : le cylindre horizontal puis vertical, le cylindre avec des ailettes incorporées dans le MCP, et enfin c'est la structure elliptique qui a été étudiée.

Mots clés : MCP, Fusion, Modélisation, Enthalpie, Volumes finis, Stockage latent, Changement de phase solide liquide

Summary:

Thermal energy storage (TES) have recently attracted considerable interest, because of its application in various fields, such as indoor spaces heating, thermal waste exploitation, water heating, cooling and air conditioning ... The use of a latent heat storage (LHS) system using phase change materials (PCM) is an efficient way to store thermal energy, and has the advantage of high storage density, in addition to the isothermal nature of the storage process. There are a large number of PCMs that change their phases at a wide range of temperature, making them attractive in a large number of applications. The high energy storage density significantly reduces the storage volume, and the relatively constant storage temperature avoids fluctuations during storage and retrieval operations; however, the main limitation when using these materials is the low thermal conductivity, and consequently, technics should be developed to improve the heat transfer for any application of LHS system.

Because of the complexity of the mathematical governing equations, due mainly to the non-linearity of the convective term in the Navier-Stokes equation, and to the viscous diffusion term related to momentum transfer in which second-order derivatives are introduced; the rigorous resolution of the problems, involving phase change process coupled with natural convection, are possible only for a restricted number of simple cases, which makes the modeling of these phenomena of a capital utility.

In this thesis we are interested in the modeling of liquid solid phase change in the presence of natural convection in different encapsulations; the physical model used is based on the enthalpy formulation, which is developed to highlight the coupling between natural convection and the melting process of the PCM. The numerical scheme used is based on the finite volume method. The effect of the geometric parameters of the capsules on the MCP melting process in the different encapsulations is highlighted.

During these investigations, the geometric shapes modeled are: the horizontal and vertical cylinder, the cylinder with fins incorporated in the PCM, and finally it is the elliptical structure that has been studied.

Key words: PCM, melting, Modeling, Enthalpy, Finite volumes, Latent storage, Liquid solid phase change

ملخص:

لقد اجتذب مجال تخزين الطاقة الحرارية مؤخرًا اهتمامًا متزايدًا، نظرًا لتطبيقاته المتعددة في مختلف المجالات، مثل تسخين الفضاءات الداخلية للمباني، استغلال المخلفات الحرارية، تسخين المياه، التبريد، وتكييف الهواء ... إن استخدام نظام تخزين الحرارة الكامنة باستخدام المواد ذات الطور المتغير هو وسيلة فعالة لتخزين الطاقة الحرارية و من مزاياه كثافة التخزين العالية، بالإضافة إلى المجال الضيق للحرارة أثناء عملية التخزين و الإفراغ.

يوجد عدد كبير من المواد المتغيرة الطور التي تغير حالتها في نطاق درجة حرارة واسعة، مما يجعلها جذابة في عدد كبير من التطبيقات. يمكن ارتفاع كثافة تخزين الطاقة من تقليل حجم التخزين بشكل كبير. كما أن ثبات درجة حرارة التخزين تجنب التقلبات أثناء عمليات التخزين والاسترجاع؛ ومع ذلك، فإن المشكلة الرئيسية مع استخدام هذه المواد، والتي تثير الكثير من الاهتمام، هي أن معظم هذه المواد لديها موصلية حرارية منخفضة، وبالتالي فإن البحث في تقنيات تمكن من تحسين نقل الحرارة ضروري لأي تطبيق لتخزين الحرارة الكامنة.

نظرًا للتعقيد الكبير في المعادلات الرياضية التي تحكم النموذج الفيزيائي، والذي يرجع بالأساس إلى الطرف الغير خطي للحمل الحراري في معادلة "نافيير ستوكس"، وكذا للمشتقة الثانية للجزء المتعلق بنقل كمية الحركة؛ فإن حل هذه المعادلات بالطريقة التحليلية غير ممكن إلا لعدد قليل من الحالات السهلة. الأمر الذي يجعل من النمذجة الرقمية أداة جد أساسية.

من خلال هذه الأطروحة نحن مهتمون بنمذجة ومحاكات تغير الطور سائل صلب مع وجود الحمل الحراري الطبيعي وذلك داخل حاويات ذات أشكال هندسية مختلفة. ويستند النموذج الرياضي المستخدم بالأساس على صياغة طاقة (اونتالبيك)، والتي تم تطويره لتسليط الضوء على الاقتران الموجود بين الحمل الحراري الطبيعي وعملية ذوبان الجسم. ويستند النظام العددي المستخدم اساسا على طريقة (الحجوم المحدودة). تبرز الدراسة بشكل جلي تأثير المعاملات المرتبطة بالأشكال الهندسية على عملية ذوبان الجسم ذو الطور المتغير من خلال هذه المحاكات، وذلك على نماذج متعددة: الاسطوانة الأفقية والرأسية، الاسطوانة مرفقة بصفائح معدنية مدمجة بداخل الجسم ذو الطور المتغير، وأخيرا الأسطوانة البيضاوية الشكل.

الكلمات المفتاحية: المواد ذات الطور المتغير، ذوبان، النمذجة، إثنالبي، أحجام محدودة، التخزين الكامن، تغيير الطور سائل صلب.

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General Introduction:

Global warming has become a major scientific and political issue during past decade. Observations since the 1950s have seen unprecedented climate changes over the last millennia; especially 2016 was the warmest year since the beginning of the surveys in 1880 with an average global warming above the $1.0\text{ }^{\circ}\text{C}$. this situation is caused by the huge amounts of greenhouse gases directly rejected into the atmosphere, leading to an unprecedented situation, which is accompanied in particular by the rise of the sea level, and all the influence on the human population at the scale of the earth.

The climate context is intimately linked to the energy context: it is well known that the combustion of fossil fuels has disastrous consequences on the climate and the environment. Without better technologies available for the exploitation of other energy sources, it is certain that hydrocarbon reserves will be intensively exploited regarding to the increasing global energy demand, that is why it is essential to optimize energy systems and improve their efficiency while promoting the use of clean, renewable and sustainable energy sources.

Solar energy is an attractive source of energy, since it is abundant on earth, clean, and inexhaustible: 1 km² of desert (Sahara) could develop a solar power of around 50 MWe (according to the technique used) and avoid the release of 200 000 tons of CO₂ per year. However, solar energy, by its nature, is intermittent (day / night), random (thunderstorms and cloud passages) and staggered in relation to daily or seasonal energy demand. Moreover, its exploitation requires the deployment of efficient storage systems allowing to adjust the production with the consumption.

By using adequate equipment sun irradiation energy can be converted into thermal and electrical energy. Depending on the temperature of the working fluid, we can differ between the low-temperature ($T < 100\text{ }^{\circ}\text{C}$), middle-temperature ($100\text{ }^{\circ}\text{C} < T < 400\text{ }^{\circ}\text{C}$) and high-temperature solar thermal energy conversion ($400\text{ }^{\circ}\text{C} < T < 4000\text{ }^{\circ}\text{C}$): For low-temperature solar energy conversion one uses flat collectors with water and air, for middle temperature conversion one uses vacuum collectors and collectors with concentrators, and for high-temperature conversion one uses solar furnaces and concentrating solar power (CSP) plants.

Solar energy can be stored in electrical batteries: the conversion of solar energy into electrical energy is assured by the use of photovoltaic panels; however, the storage of electrical energy in batteries presents the weak point of this technology since it increases the cost of the system and reduces its use.

Thermal energy storage (TES) are attractive in many thermal practical applications like in water and space heating, air-conditioning and cooling... . Implementing a thermal storage system often requires compromises on storage capacity, power delivered or stored,

and losses. Depending on the physical phenomenon involved (heating, endothermic / exothermic chemical reactions, or phase change), thermal storage systems are given in three different forms:

The storage by variation of sensible enthalpy: this is the most used way currently. It is a non-isothermal process in which the energy absorbed / released by the storage medium is proportional to the variation in temperature of the heat transfer fluid. The quantity of stored thermal energy is expressed in the following relation: $\Delta E = m \cdot cp \cdot \Delta T$

Where: m is the mass [kg], cp the mass calorific capacity at constant pressure [J / (kg K)] and ΔT the temperature difference [K] between an initial state and an end state.

Thermochemical storage: This type of storage relies on two types of reactions: reversible endothermic chemical reactions and sorption reactions.

Latent heat storage systems (LHS) using phase change materials (PCMs) are very effective in storing thermal energy, since they have a high-energy storage density in addition of the isothermal nature of the storage process. They find a lot of applications like in solar engineering, heat pumps, and spacecraft thermal control applications. The wide range of melting temperatures for PCMs make them very attractive in a lot of applications, their higher energy storage density reduces considerably the storage volume, and a relatively constant storage temperature avoids fluctuation during storage and retrieval operations; but the major issue which has to be addressed is the low thermal conductivity that most phase-change materials (PCM) have, so enhancement techniques are very useful for any latent heat thermal storage (LHTS) applications.

A latent thermal storage system possesses three major components: (i) a heat storage system that undergoes a solid-to-liquid transformation within the desired operating temperature range, (ii) containment of the storage substance, and (iii) a heat exchanging surface for transferring heat from the source to the storage substance and from the latter to the heat load. Solid-liquid phase change heat transfer relevant to latent heat-of-fusion energy storage systems has been a subject of many theoretical and experimental studies.

The aim of this thesis is to study the heat transfer process in phase change materials in terms of different encapsulation geometries to effectively charge and discharge latent heat energy. Modelling and simulation of phase change material are investigated to assess the effects of different parameters.

The first chapter introduces the physical modeling and mathematical formulation of the enthalpy porosity approach for heat transfer problems associated with melting and

solidification of phase change material. One-dimensional Stefan Problem for Pure Materials is then presented.

Chapter 2 is devoted to a rectangular cavity in which the melting process is studied. Firstly, the problem of Gau et al is presented, then the rectangular cavity is studied in term of the inclination angle and the effect on this angle on the melting process of phase change material is investigated, results for different inclination angles are presented and discussed. Melting of PCM over wavy surface is then studied; it aims to investigate the amplitude, and the number of wavy surface effects on the rate of melting of a square wavy cavity. Both horizontal and vertical wavy surfaces are studied and their results are compared to flat surface. They show that wavy surfaces are more attractive in heat transfer because of their capability to facilitate the fluid motion near the wavy surface.

Chapter 3 is devoted to study the PCM's melting process in cylindrical enclosure, the first part treat the vertical cylinder filled with PCM; it aim to study the natural convection dominated melting of a PCM within a cylindrical capsule in its vertical position. The second part is concerned in the melting of PCM in a horizontal cylinder: this study aims to the numerical investigation of the natural convection dominated melting of the material within a horizontal cylindrical capsule for Rayleigh numbers in the range from 10^4 to 10^8 . The enthalpy-porosity method is used, where the solid-liquid interface is implicitly determined in the calculation domain. This chapter is ended by studying the effect of introducing fins embedded in PCM in horizontal cylinder, the aim of this section is to point out some of the natural convection characteristics during the melting process of PCM around a circular cylinder with three rectangular fins. The contribution of the present study to the actual knowledge lies in the fact that the problem of the melting process is formulated using the enthalpy-porosity based method.

Chapter 4 investigates the case of PCM Melting around Tree horizontal cylinders; this investigation aims to understand the heat transfer process during the charging phase of TES system which uses the latent heat-of-fusion of PCM. In this section we are searching for the optimum arrangement of the tubes through which the working fluid is circulated, for an effective utilization of the PCM.

In chapter 5 the melting process in an elliptical enclosure filled with PCM is investigated, the results show that an increase in the aspect ratio has a positive effect on the melting rate of the PCM. The main conclusion of this chapter is that the aspect ratio of the elliptical enclosure has a great effect on the melting time. Therefore, the ellipse's ratio can be utilized to regulate the melting process in elliptical capsules.

Chapter I:

Physical Modeling of Solid-Liquid Phase Change Material.

1. Introduction:

Heat transfer problems, involving solidification and melting of pure phase change material (PCM), are facing an increasing interest in view of their extensive occurrence in many practical applications like in solar energy units, crystal growth, industrial refrigeration, purification of metals, casting and welding ... [1- 6]. PCMs are largely used in thermal control and storage systems. The thermal energy is stored during low energy demand periods, and then, released during high energy demand periods [7-9].

There is no doubt that several studies have approaches the significant processes taking place during melting and freezing of pure material both from numerical and experimental point of view [7-10], however the non-linearity of the governing energy equation and a wide variety of geometric structures with divers thermal boundary conditions offer a fertile ground for challenging basic research problems [10-13]. Furthermore, numerous industrial applications in different industries provide the necessary incentive for engineering research and development.

In comparison to numerical approaches, the experimental analysis can present the real behavior and performance of PCM more directly, visibly and credibly without any pre-set assumptions. however, in some cases the experiments are unachievable, such as in the large scale or in an unsteady around environment, thus the consumption of time and cost will be higher than a theoretical approach, hence, the budget and processes needs to be set up and scheduled. In addition, the experiment needs that a test rig has to be constructed examined and operated properly. Finely relevant parameters need to be monitored, measuring apparatuses need to be calibrated, and the failed data need to be eliminated.

Apart from these agonizing experimental matters, testing errors cannot be avoided entirely; however, the numerical methods can address all these deficiencies without los in determining the PCMs performance accurately. One of the most important merits of the theoretical/numerical approaches is that various conditions can be carried out by changing the variables in a numerical model.

Melting and solidification of phase change materials where a moving boundary separates two phases in phase change transitions. Many technics to solve moving boundary problems are well studied [3]. During melting and solidification processes of a pure substance, the effect of natural convection on the motion and the shape of the solid-liquid interface are clarified in these studies. Results show that natural convection in the melt region affect significantly the solid-liquid interface shape, and the rate of melting of the PCM.

Mathematical models for melting and solidification processes are classified in general in two main classes: single-domain method and multi-domain method. In multi-domain method, conservation equations are solved in each phase separately, continuity of flow and temperature conditions are then imposed at the solid-liquid interface, while in the single-domain method (fixed grid) a unique system of conservation equations and boundary conditions is used for the entire domain containing liquid and solid phases, a suitable source term is added in governing equations to account for the interface conditions. This model is identified as the enthalpy-porosity method [10 - 15]

The aim of this chapter is to present the mathematical formulation of the enthalpy-porosity method applied to heat transfer problems with melting and solidification of pure change phase materials. At first, we intend to expose the numerical procedure of the enthalpy-porosity approach and the 1D phase Stefan problem. Then, we propose to examine numerical modeling of various PCM packages' geometries.

2. Mathematical formulation of solid-liquid phase change for pure materials

As in any simulation problem, assumptions should be considered for simplification purpose; the problem of heat transfer involving solid-liquid phase-change process is not an exception. The equations governing the problem are based on the concept of a continuum model. Furthermore, each phase is supposed to be homogeneous and isotropic.

Density changes between the two phases gives consequences to be considered. Temperature variations in the liquid phase are necessarily detectable during heat transfer. In the liquid zone, natural convection currents can be sufficiently large, generating buoyancy forces for unstable situations which can produce free motions. The natural convection circulation in the liquid zone could have an important effect on the motion of phase-change interface and heat transfer. Situations where density differences are present between phases or where density variations occur in the liquid will be subject to a special attention [14-17].

Unsteady melting or solidification of PCM is governed by basic laws represented by the continuity, momentum and energy equations with the following assumptions:

- The thermo-physical properties of the PCM are constant but may be different for the liquid and solid phases.
- The liquid density variations arise only in the buoyancy source term, but are otherwise neglected: Boussinesq approximation.
- The liquid is supposed to be Newtonian.
- We neglect the viscous dissipation.
- The regime in the melt is supposed to be laminar.

Since the present formulation deals with solutions on unstructured grids, the governing conservation equations for mass, momentum and energy are represented in their respective integral forms. Considering the foregoing assumptions, the conservation laws are stated as:

$$\int_S \vec{u} \cdot \vec{n} dS = 0 \quad (1)$$

$$\frac{\partial}{\partial t} \int_V \rho \vec{u} dV + \int_S \rho \vec{u} \vec{u} \cdot \vec{n} dS = - \int_V \vec{\nabla} p dV + \int_S \vec{\tau} \cdot \vec{n} dS + \int_V \vec{A}_U dV \quad (2)$$

$$\frac{\partial}{\partial t} \int_V \rho c_p T dV + \int_S \rho c_p T \vec{u} \cdot \vec{n} dS = \int_S \lambda \vec{\nabla} T \cdot \vec{n} dS + \int_V \rho L_F \frac{\partial f}{\partial t} dV \quad (3)$$

\vec{u} Stands for the velocity vector, p for the pressure and T the temperature. For a Newtonian fluid the viscous stress tensor $\vec{\tau}$ is given by the following equation:

$$\vec{\tau} = \mu \left(\vec{\nabla} u + (\vec{\nabla} u)^T \right) \quad (4)$$

The integration is made over a control volume V surrounded by a surface S , this one is oriented by an outward unit normal vector \vec{n} . The source term in Eq.(2) is given by the following equation:

$$\vec{A}_U = \rho \beta (T - T_m) \vec{g} + A \vec{u} \quad (5)$$

β stands for the volumetric thermal expansion coefficient, \vec{g} is the acceleration of gravity vector. The buoyancy forces caused by the thermal dilatation are presented by the first part of the term source. T_m represents the melting temperature of the PCM. In order to account for the velocity switch-off imposed in the solid region, the last term is added. When solving the momentum field, the velocity at the solid phase computational cell should be canceled

while those located in the liquid phase remain unaffected. The mass in the corresponding computational cell should begin to move as the solid region melts. The Darcy-like momentum source term is adopted here to simulate the velocity switch-off [18-20]

$$A = -\frac{C_1 (1-f)^2}{f^3 + \varepsilon} \quad (6)$$

Where the constant C_1 take a large value to cancel the velocity at the cell becomes solid, and ε is small number used here to prevent the division by zero when the computation is done on the solid region: $f = 0$. In this work, $c = 1 \times 10^5 \text{ Kg} / \text{m}^3$ and $\varepsilon = 0,001$ are used.

3. Numerical procedure:

The resolution of the problem consists in implementing the conservation equations in an in house code; excellent deal was found in comparison the results obtained by the present formulation with those of the literature [11].

This model consists of a two dimensional unstructured finite-volume framework applied to hybrid meshes; we use the new source algorithm proposed by Voller [17] to calculate the source term of the energy equation.

To solve for the Eqs.1-3 we implement theme in an in house code, based on a three dimensional unstructured finite-volume framework applied to hybrid meshes. The values of the variables are stored in cell centers in a collocated arrangement.

The conservation equations have all the same next general form:

$$\frac{\partial}{\partial t} \int_V \rho \phi dV + \sum_i \int_{s_j} \left[\rho u_i \phi - \Gamma_\phi \frac{\partial \phi}{\partial x_i} \right] dS_i = \int_V S_\phi dV \quad (7)$$

In this equation we can see four distinctive parts: rate of change, convection, diffusion and source term (the diffusion term is absent in the equation of conservation of mass). The source term and the rate-of-change are integrated over the cell volume, whereas the convection and diffusion terms constitute the sum of fluxes through the control volume faces.

Since solidification and melting are generally transient phenomena, and due to stability limitations, the explicit schemes are not used here: implicit schemes are preferred and the first order Euler scheme is our simplest choice. The surface and volume integrals are calculated implicitly at the new time level in this method. The value of the variable ϕ was

calculated at the center of the control volume (CV). Therefore, the time integral is expressed as:

$$\frac{\partial}{\partial t} \int_V \rho \phi dV \approx \frac{(\rho \phi V)_P^{n+1} - (\rho \phi V)_P^n}{\Delta t} \quad (8)$$

Where n and $n + 1$ the old and the new time levels.

The convection flux of the quantity, ϕ , can be approximated over the CV surface as:

$$\int_{S_e} \rho \phi u_i dS_i \approx \sum_i (\rho u_{e,i} n_{e,i} S_e)^{n+1} \phi_e^{n+1} \quad (9)$$

An appropriate interpolation is needed for the face values ϕ_e for accurate propose. In our case we can use one of three non-linear high-resolution bounded schemes Quick, Smart and Cubista [21-23], where ϕ_e is expressed as a function of the centered differencing (CD) and its upwind (UP) value:

$$\phi_e = \phi_e^{UP} + \gamma (\phi_e^{CD} - \phi_e^{UP}) \quad (10)$$

Using the normalized variable diagram (NVD) framework and observing the convection boundedness criterion (CBC)[24], the coefficient γ is calculated for each face based on the flow solution local shape. The first term of the RHS of Eq.(10) is accounted for implicitly, whereas the second term is treated explicitly with the deferred-correction practice. The convective terms can also be handled by any of the following five schemes: upwind, centered hybrid, power law and exponential.

The diffusive flux ϕ , on the other hand, may be approximated over the CV as:

$$- \int_{S_e} \Gamma_\phi \frac{\partial \phi}{\partial x_i} dS_i \approx - \sum_i (\Gamma_\phi \frac{\partial \phi}{\partial x_i} n_i)_e S_e \quad (11)$$

The orthogonality is an exception in the case of unstructured meshes. In this case the normal gradient is decomposed into an implicit part using the values of ϕ at the centers of the two cells sharing the face 'e' and a non-orthogonality correction term treated explicitly by a deferred approach to preserve the second-order accuracy of the centered differencing [25].

The equation may also be written in a conventional manner as

$$A_p \phi_p = \sum_{nb} A_{nb} \phi_{nb} + b_\phi \quad (12)$$

The neighboring CVs contribute in the coefficients A_{nb} . On the other hand, the central coefficient A_p , contain the contributions of the transient term and all the neighbors.

The iterative SIMPLE algorithm developed by Patankar and Spalding [26-28] is used to couple the dependent variables.

The new source algorithm developed by Voller [29] is used in this study to update the liquid fraction f as follows:

$$f_p^{n+1} = f_p^n + \frac{a_p^n \Delta t}{\rho L_f V} (T_p - T_m) \quad (13)$$

which is followed by an the correction:

$$f_p^{n+1} = \begin{cases} 0 & \text{if } f_p^{n+1} < 0 \\ 1 & \text{if } f_p^{n+1} > 1 \end{cases} \quad (14)$$

An ILU-preconditioned GMRES procedure implemented in the IML++ library [30] is used to solve for the momentum, temperature, and pressure correction. The open-source software Gmsh [31] is also used to generate all of the computational meshes.

The computations were performed on the personal Pentium PC. The post-processing of the data was conducted by using a commercial data visualization software package, TECPLOT, version 10 for 1-D and 2-D data presentation. The platform for TecPlot data processing was a Pentium PC.

4. One-dimensional Stefan Problem for Pure Materials

When heat transfer equations are to be considered in the melting phase only, the problem is qualified as “One-phase Stefan problem”: the heat transfer involves phase change in medium which is initially at its melting temperature and remains in thermal equilibrium during the phase change process. Heat transfer equations are written in the formed phase only where thickness changes with time according to the problem conditions. When heat transfer equations are to be considered in the solid and the liquid phases, the problem is referred as “two-phase Stefan problem” [32-35].

In one-phase Stefan problem [36], a surface of a semi-infinite sheet of PCM, initially ($t=0$) at the melting temperature (T_m), is subjected to a temperature T_w lower than T_m , at $x=0$; and this temperature is maintained for all times $t > 0$. Instantly the solidification starts and an interface solid-liquid appears at the position ($x = 0$), the thickness of this interface grow with time; the challenge is to determine the location of the moving interface: $S(t)$ and to calculate the temperature distribution $T(x,t)$ in the solid region ($0 < x < S(t)$): Fig. 1

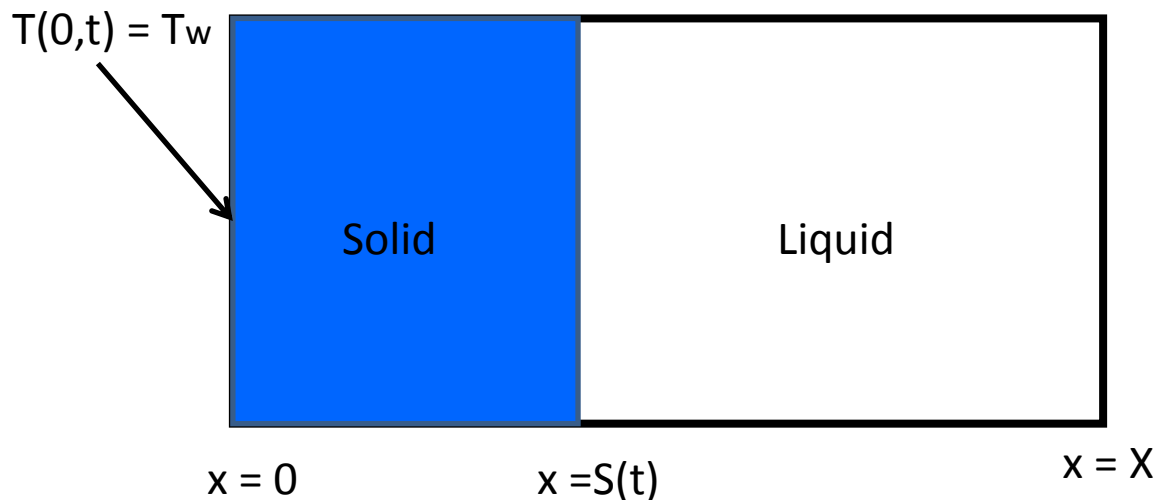


Fig.1: The one-phase Stefan problem schematic

The nonlinearity of the thermal energy equation at the solid/liquid interface makes the solution of such problems inherently difficult. To overcome the difficulty, we assume uniform and equal properties for both phases. Thus, the governing equation for this problem will be

$$\rho c \frac{\partial T(x,t)}{\partial t} = k \Delta T + \rho c \frac{\partial f_s}{\partial t} \quad (15)$$

If the cold source is a plate, a cylinder or a sphere, we will use Cartesian, cylindrical, or spherical coordinates. Eq.(15) will be respectively

$$\rho c \frac{\partial T(x,t)}{\partial t} = \frac{k}{x^n} \frac{\partial}{\partial x} \left(x^n \frac{\partial T}{\partial x} \right) + \rho c \frac{\partial f_s}{\partial t} \quad (16)$$

Where $n = \begin{cases} 0; \text{Cartesian} \\ 1; \text{Cylindrical} \\ 2; \text{Spherical} \end{cases}$ and f_s is the solid fraction.

In [35], Neumann gives the analytical solution for the problem as:

$$T(x,t) = T_0 + B \cdot \text{erf} \left(\frac{x}{2\sqrt{\alpha t}} \right) \quad (17)$$

Where $B = \frac{T_0 - T_m}{\text{erf}(\xi)}$, α is the thermal diffusivity and ξ is the root of the transcendental equation:

$$\xi e^{\xi^2} (\text{erf} \xi) = \frac{c_p (T_0 - T_m)}{L_F \sqrt{\pi}} \quad (18)$$

The location of the interface in time is described by the equation

$$S(t) = 2 \xi \sqrt{\alpha t} \quad (19)$$

The last equation allows us to test the enthalpy-porosity method and our algorithm described in paragraph 3. To appreciate the nature of the numerical solution, we consider a problem which is defined by the material parameters and initial and boundary conditions are given in Table 1.

$X = 0.5 \text{ m}$	$\rho = 1000 \text{ kg / m}^3$
$T(0,t) = -20 \text{ }^\circ\text{C}$	$c_p = 2200 \text{ J / (K.kg)}$
$TW = -20 \text{ }^\circ\text{C}$	$k = 1.0 \text{ W / (K.m)}$
$T_m = 5.3 \text{ }^\circ\text{C}$	$L_F = 232000 \text{ J / kg}$

Table 1: Parameters used in Stefan problem

Using the enthalpy-porosity approach, the position of the interface in time can be calculated by the following expression:

$$S(t) = (1 - \sum_{i=1}^N f_{s,i}) X \quad (20)$$

Where N is the number of the control volume (CV) in the PCM domain.

In this problem, 80 CV is used and the time step is $\Delta t = 1 s$. The material parameters given in Table 1 define a Stefan number of

$$Ste = \frac{c_p (T_m - T_w)}{L_F} = 0.24 \quad (21)$$

The evolution temperature at the position $x = 0.25m$ is drawn, together with the front position in Fig.2. As we can see, the results produced by the analytical and enthalpy-porosity methods are hardly distinguishable.

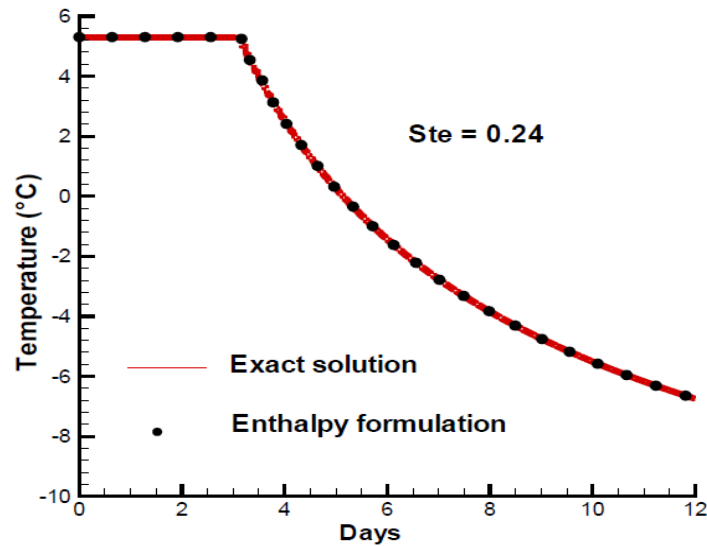


Fig.2 (a): Temperature at $x = 0.25 m$

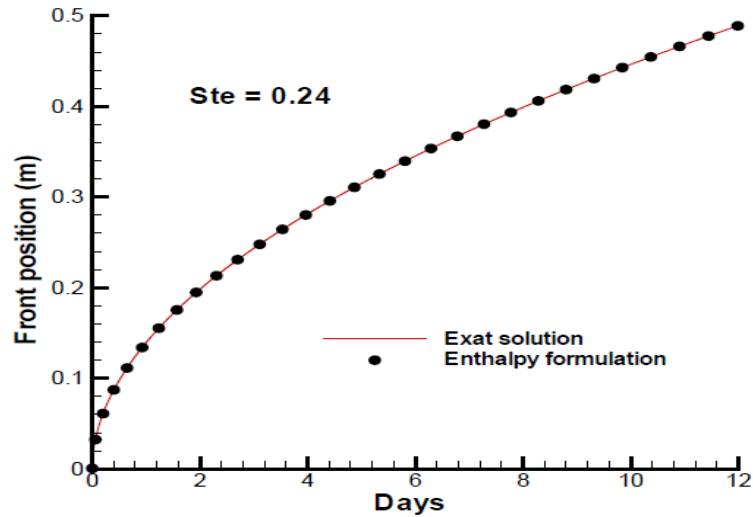


Fig.2 (b): Front position versus time

The same investigation was next performed for two different Stefan numbers: $Ste = 0.275$ and $Ste = 0.36$. The two cases were generated by increasing the latent heat whereas all other material, boundary and mesh data remain unchanged. It is clearly seen that as the Stefan number increases, the interface moves faster (see Fig.3). The computations are carried on even after the complete solidification of the domain; hence, the interface location appears to remain stationary at 0.5m.

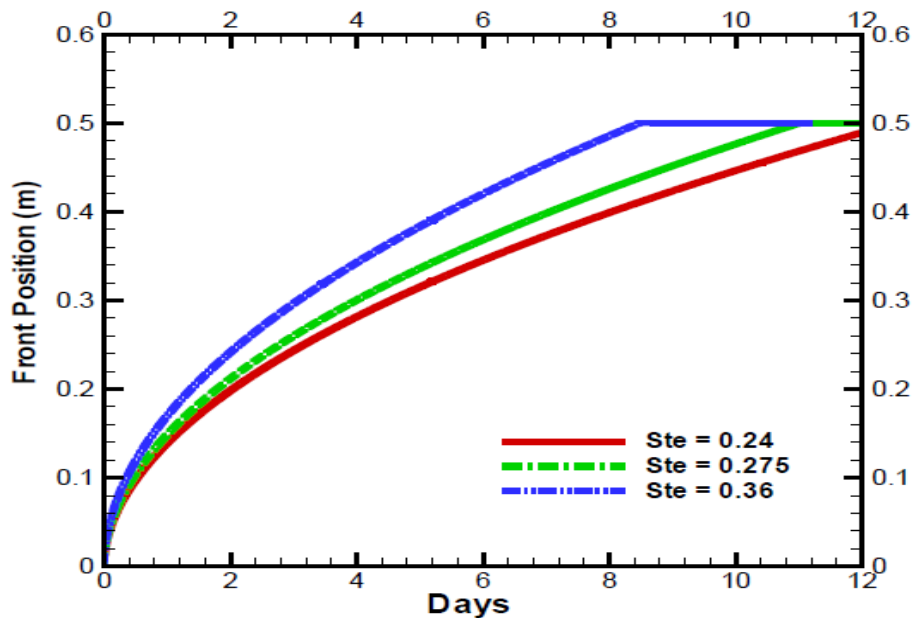


Fig.3: Effect of the Stefan number on the front position

5. Conclusion:

In this chapter we have introduced the principle of the physical modeling of Solid-Liquid Phase Change material. Mathematical formulation of the enthalpy-porosity method applied to heat transfer problems with melting and solidification of pure change phase materials was presented. At first, we exposed the numerical procedure of the enthalpy-porosity approach for the 1D phase Stefan problem. Then, we propose to examine numerical modeling of various PCM packages' geometries; the governing conservation equations for mass, momentum and energy are represented. Hence implicit schemes are often preferred and the simplest choice is the first order Euler scheme.

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Chapter II :

Melting of Phase change Material in rectangular cavity

1- Introduction:

The solid-liquid phase change process is of great importance, it is intensively used in thermal energy storage systems where a temporal phase shift difference exists between the supply and the utilization of this energy. The continuity of a thermal process in energy systems is assured by latent energy storage process such as in solar thermal applications like, heating, cooling, hot water, air- conditioning, etc., because of its intermittent nature. In such applications, a LES system should store the energy absorbed as long as possible, and supply this energy on cloudy days when the production is low. In order to predict accurately the storage system performance and avoid overdesign of the system, good understanding of the melting process is needed. Natural convection heat transfer in different geometrical capsules finds various practical applications. When studying the solidification process, the sole heat transport mechanism present is the conduction, but when melting is considered, natural convection appears in the melt region, thus an enhancement in the heat transfer rate is appeared in comparison to the solidification process.

2- The problem of Gau et al:

Because of its wide-ranging engineering applications in many fields like in thermal energy storage, casting, and metallurgy ..., melting of phase change materials in rectangular enclosures has received considerable attention. It has been extensively studied both experimentally and numerically [1-27], especially the problem of gallium melting in a rectangular cavity heated from one side has received considerable attention. In this problem solid gallium contained in a rectangular cavity with one of its vertical walls suddenly elevated to a higher temperature, while the other vertical wall is maintained at a low temperature below the freezing point, the other horizontal boundaries are kept insulated thermally (Fig.4).

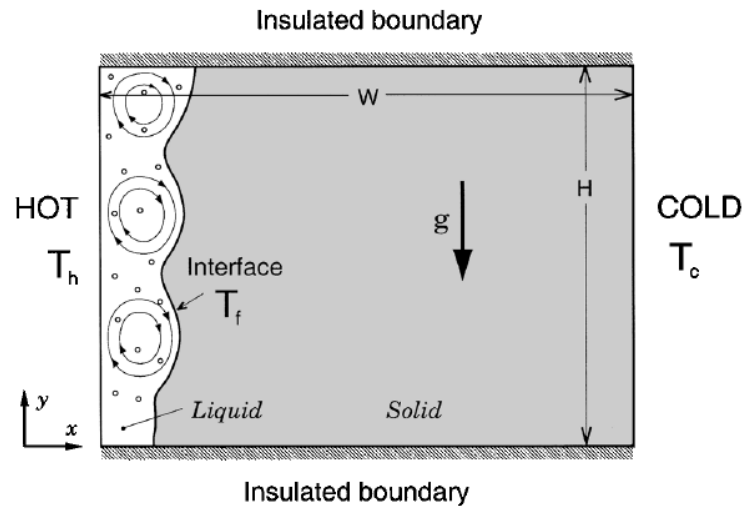


Fig.4: Melting of phase change materials in rectangular enclosures

At the initial stage, a thin melt layer forms in contact to the heated surface proving that the predominant mode of heat transfer in this stage is conduction. As time progresses, the buoyancy forces induces a flow due to temperature gradient causing the melt volume at the top to recede rapidly compared to the bottom of the enclosure leading to a deformation of the originally planar solid-liquid interface

A lot of studies have been done on gallium melting in two dimensional rectangular cavities problem for comparison purposes, the accuracy of their results has never been clearly proofed, the global features of the solution is a subject of disagreement for researchers: the cells number observed during the melting process in the fluid flow present a point of contention. Hale and Viskanta [21] were the first experimental studies on melting inside rectangular cavity taking account of the convective effects. They had the objective to evidence influences of the natural convection on the melting process inside that cavity.

Gau and Viskanta [22] realized some experiments on melting and solidification processes of gallium inside a rectangular cavity having one of the vertical walls heated by a temperature above the melting temperature. Correlations have been proposed by the authors between some dimensionless parameters and the volume of the molten metal. To enable a consistent analysis of the of natural convection influence on the melting and solidification processes, temperature profiles were plotted.

Brent et al [23] investigated an enthalpy-porosity mathematical formulation based method which allows the use of a fixed spatial grid. The power law discretization scheme was chosen to be a 42×32 grid, and a time step ranging from 5 to 10s; the thermo-physical properties of gallium were supposed constants, except for the density, where the Boussinesq

approximation was applied. The results showed that the flow was characterized by a single roll structure that grew in size as melting proceeded. The authors compared their results with those of Gau et al. [22]: their results show an agreement between the computed and experimental liquid – solid interface, unlike what was found by Webb and Viskanta [24] where the interface location on their plots reveals as much as 20% discrepancy between numerical and experimental results. Dantzig [25] found a melt flow structure containing multiple small rolls that disappears at longer times, whereas only a unique roll had been reported in Brent's previous study; on the other hand, the findings of Dantzig were in disaccord with previous experimental results, which led him to share his question: "Is the multiple cell solution or the single cell solution correct".

Using the FIDAP commercial code, Dantzig state that the reason for the failure of earlier studies in predicting a multicellular flow structure, were the upwinding and too coarse grids: earlier in the melting process, the treatment of the convective terms by the upwinding scheme is the raison of eddies suppression observed with a centered scheme. Unfortunately his code failed to converge when coarser grids were employed. Stella and Giangi [26], confirmed the work of Dantzig when they use finer grids (several times finer comparing to earlier studies) along with the centered discretization scheme. Cerimele et al. [27] suggested that numerical simulations realized on the gallium melting were not consistent for the experimental procedure; this one involved a pour-in/pour-out procedure that resulted in an actual restart of the fluid in the melt from uniform temperature and zero fluid velocity. Calentano and Cruchaga [28] confirmed earlier results found by Dantzig by performing numerical investigations showing two rolls, early during the melting process and three later.

More recently, Wintruff and Gunther [29] found in their numerical results an initial roll structure with four rolls, which match very well those of Stella et al. [26], but they did not provide details for the numerical parameters.

A comparison of four discretization schemes (upwind, hybrid, centered, power law) for the problem of gallium melting, was presented by Kim et al. [30], they didn't find significant differences between results for the various schemes which are all showing a unique roll since they used too coarse grids. The Streamline Upwind/Petrov Galerkin finite element method combined with a fixed grid primitive variable method was used by Gong and Mujumdar [31] in their study for melting of a pure PCM in a rectangular container. The validation was done by comparing their results with those obtained by Gau and Viskanta [22] and Lacroix [32].

In order to clarify the controversy over gallium melting inside a rectangular cavity heated from the side, Hannoun et al. [33] investigated the effect of some parameters (grid-converged solution, grid sizes, discretization schemes) on the flow structure. They tested three discretization schemes (upwind, hybrid and centered) and different grid sizes. The main conclusion was that flow structure is multicellular for this problem.

In [34] Vidalain et al. presented a numerical model based on the conduction equation for the solid and liquid phases applicable to phase change problems. This model simulates the liquid solid interface and the overall system thermal behavior without resolving the Navier-Stokes equations. They take into account the natural convection effect by the use of an enhanced thermal conductivity which is function of the dimensionless numbers and the geometry of the flow. Very good agreement with those of the CFD model was found for the model proposed.

3- Inclination angle effect on melting process of phase change material in a rectangular cavity:

a. Introduction

In order to analyze the effect of natural convection on the melting process of the PCM, Various investigations have been carried out [6-10], but few studies have been consacred to the effect of the inclination angle on melting process of PCM in an enclosure like in [11-13]. Webb and Viskanta [11] studied experimentally the melting heat transfer of n-octadecane in an inclined rectangular enclosure. During the experience the only recorded parameter was the interface shapes which were then used to infer the structure of the flow. The main conclusion was that the decreasing of the inclination angle causes the increases of non-uniform melting of the solid PCM; this is caused by the three-dimensionality of the flow field.

Another experimental investigation was performed by Akgun et al. [12] witch concern the melting and solidification process of paraffin in a vertical annular enclosure. It was found that when the enclosure is tilted by 5° from its vertical position the melting time decreased by 30%. Sharifi et al. [13] investigated the effect of tilting a vertical warm cylinder during the outward melting. Their experiments were performed for small inclination angles of 5° and 10° . They conclude that small tilting of the enclosure affects significantly the temperature distribution inside the PCM, and the temporal evolution of the solid-liquid interface with a

three-dimensional shape resulting from the interaction between 3D convection currents in the liquid PCM with the solid interface.

A numerical study of the melting process with natural convection in an inclined cavity using the enthalpy-based lattice Boltzmann method was performed by Jourabian et al. [14]. The study was carried out for Stefan number of 10, Rayleigh number ranging from 104 to 106, and inclination angle ranging from -30° to $+30^\circ$. The results indicated that an increase in Rayleigh number results in intensifying the melting rate at each inclination angle.

Recently an experimental investigation was performed by Kamkari et al. [15]: the investigation of the heat transfer process and melting behavior for lauric acid (with high Prandtl number) was done for different inclination angles on a rectangular enclosure. The main conclusion was the enhancement on the heat transfer ratio for the horizontal enclosure by more than two times higher than that of the vertical enclosure.

This section investigates the heat transfer process and melting behavior of gallium (as a low Prandtl number PCM) in a rectangular cavity for different inclination angles. The enthalpy-porosity based method was chosen for the formulation of the problem. A numerical code is developed based on unstructured mesh and a finite-volume method.

b. Results and discussions :

The effect of inclination angle on the melting process inside a rectangular encapsulation is carried out by means of the exploration of the flow and temperature fields. In this numerical study, the rectangular cavity is chosen to be **120 mm** in height and **50 mm** in width (see Fig.5) and containing solid Gallium initially at temperature $T_i = 15^\circ$.

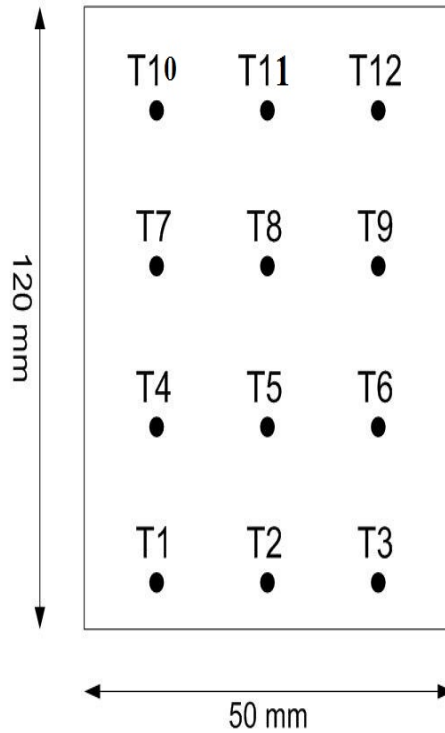


Fig.5: rectangular cavity filled by PCM

Table 1 resume thermo-physical properties of the Galium for which the Prandtl number this temperature is 0.0216.

Density (liquid)	6093 kg.m^{-3}
Reference density	6095 kg.m^{-3}
Reference temperature	$29.78 \text{ }^\circ\text{C}$
Volumetric thermal expansion coefficient of liquid	$1.2 \cdot 10^{-4} \text{ K}^{-1}$
Thermal conductivity	$32 \text{ W.m}^{-1}.\text{K}^{-1}$
Melting temperature	$29.78 \text{ }^\circ\text{C}$
Latent heat of fusion	80160 J.kg^{-1}
Specific heat capacity	381.5 J.kg^{-1}
Dynamic viscosity	$1.81 \cdot 10^{-3} \text{ kg.m}^{-1}.\text{s}^{-1}$
Prandtl number	$2.16 \cdot 10^{-2}$

Table 2: Thermo physical Properties of Pure Gallium

The right wall of the cavity was maintained at constant temperature $T_B = 35^\circ C$. The other walls are considered to be adiabatic. This numerical study was done using 92000 cells, and the time step was chosen to be 10^{-4} s, which is sufficient to give accurate results.

In Figs.6, 7 and 8, are presented the stream lines, the temperatures contours and the liquid-solid interfaces at times 300s, 800s and 1300s for the three inclination angles 90° , 60° and 30° of the vertical cavity. The results show that for both the three studied inclination angles, in the first stages, heat transfer in the melt zone is predominated by conduction and the liquid-solid interface takes the wall's profile. This mode prevails as long as the viscous force opposes the fluid motion; in this situation the solid-liquid interface is uniform and parallel to the heated wall. However, as melting progresses, the melting zone expands and convection takes over conduction. The rate of melting is higher near the top of the solid-liquid interface where warm fluid, after being heated by the hot wall, impinges the solid PCM. As the fluid touch relatively cold interface, the liquid cools.

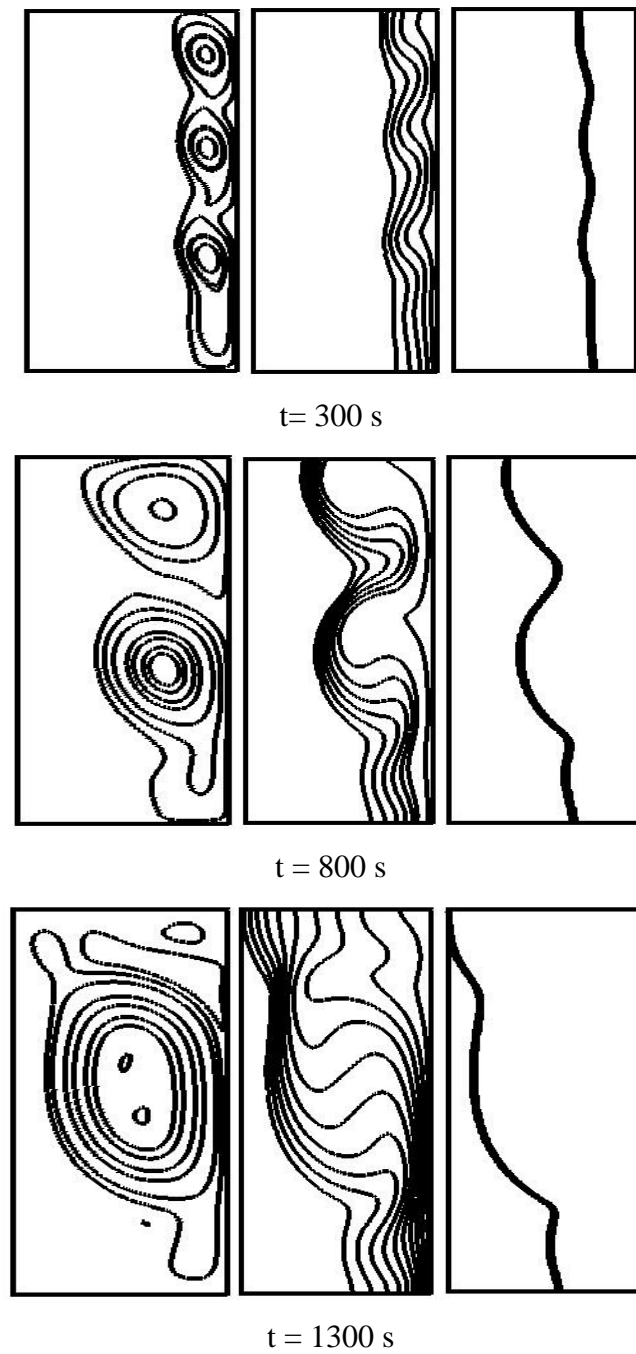


Fig.6: Stream lines, temperatures contours and interfaces liquid-solid at times 300s, 800s and 1300s (Inclination angle = 90°)

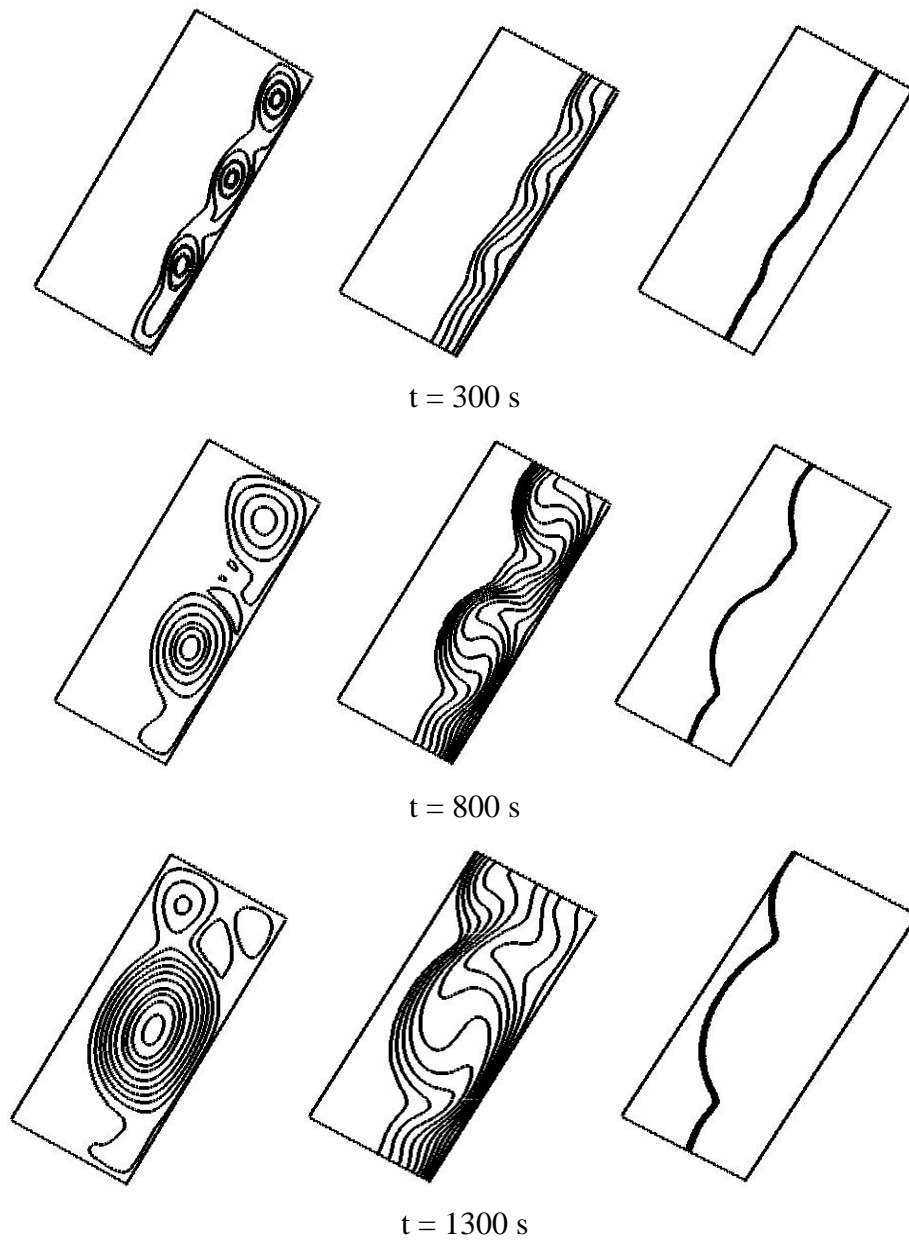


Fig.7: Stream lines, temperatures contours and interfaces liquid-solid at times 300s, 800s and 1300s (Inclination angle = 60°)

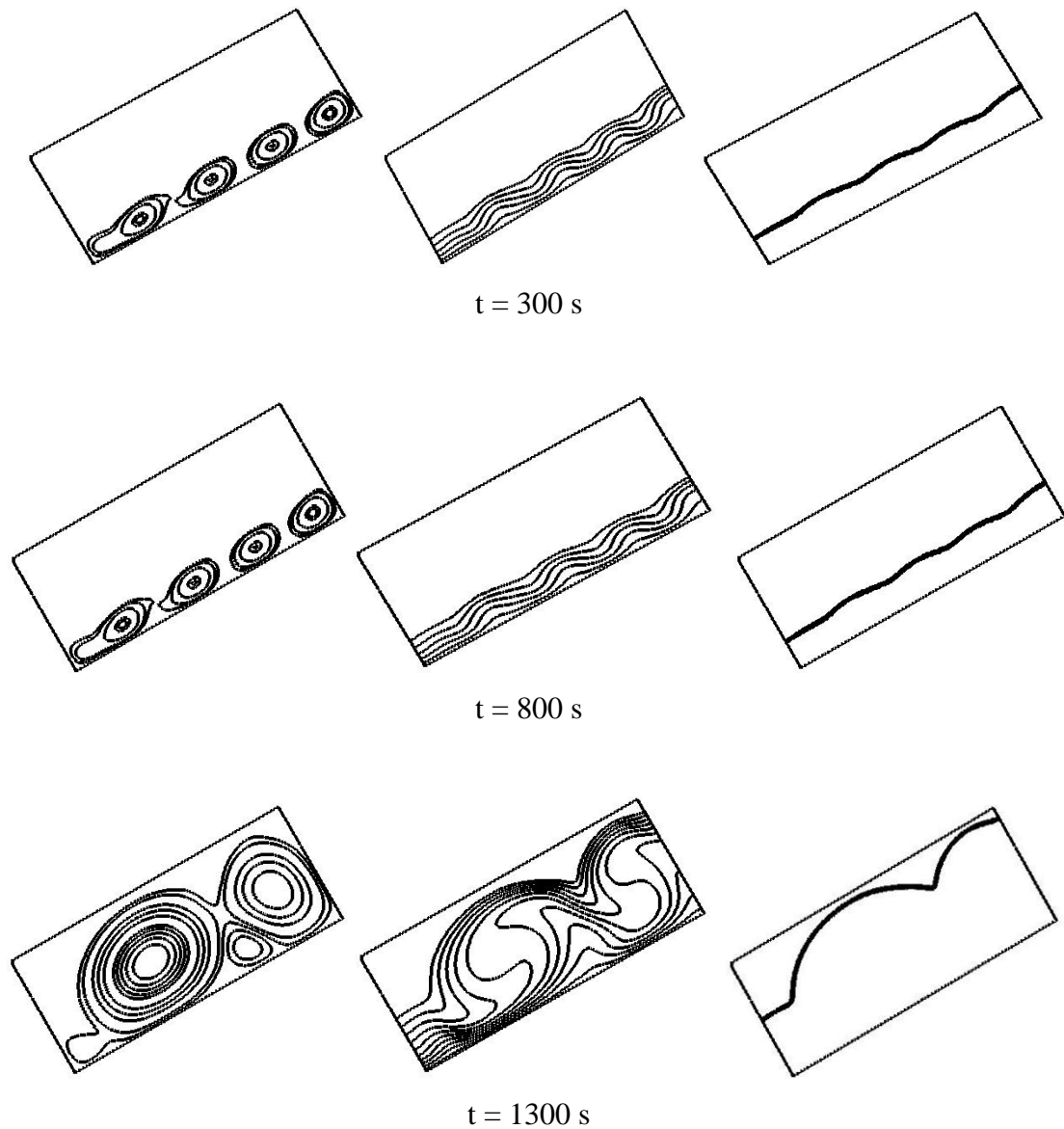


Fig.8: Stream lines, temperatures contours and interfaces liquid-solid at times 300s, 800s and 1300s (Inclination angle = 30°)

We use a two dimensional model in the present study, however, the duration of the three dimensional convection is very short compared to the whole melting process. In the case of the inclination angle of 0° the results are presented in Fig.9.

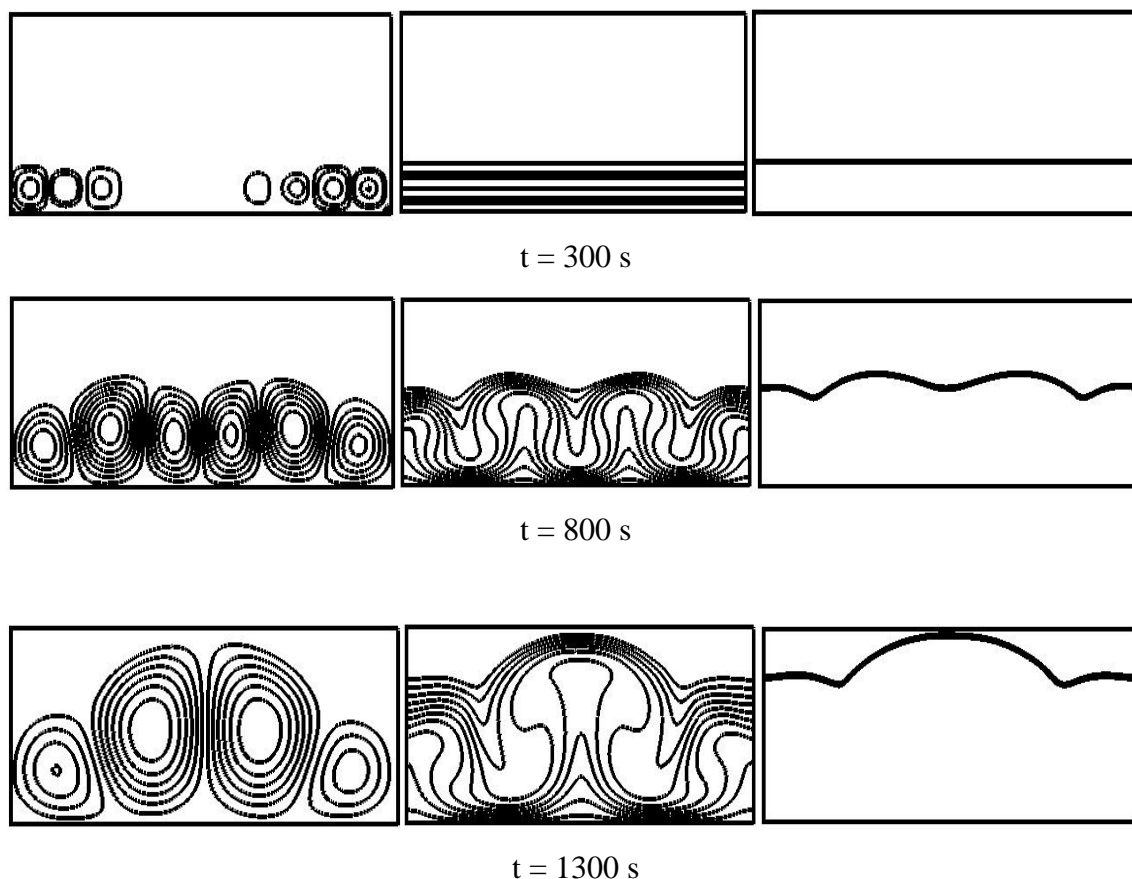


Fig.9: Stream lines, temperatures contours and interfaces liquid-solid at times 300s, 800s and 1300s (Inclination angle = 0°)

After the conduction dominating stage, convection takes place and a complex structure of the fluid appears, characterized by a multi-cellular flow patterns. The number and the size of the Bénard cells depends on the height of the molten phase. As convection develops, the liquid zone increases and the cells at the center disappear while those at the bottom corners increase.

The temperature history at different positions inside the cavity for each inclination angle is presented in Fig. 10.

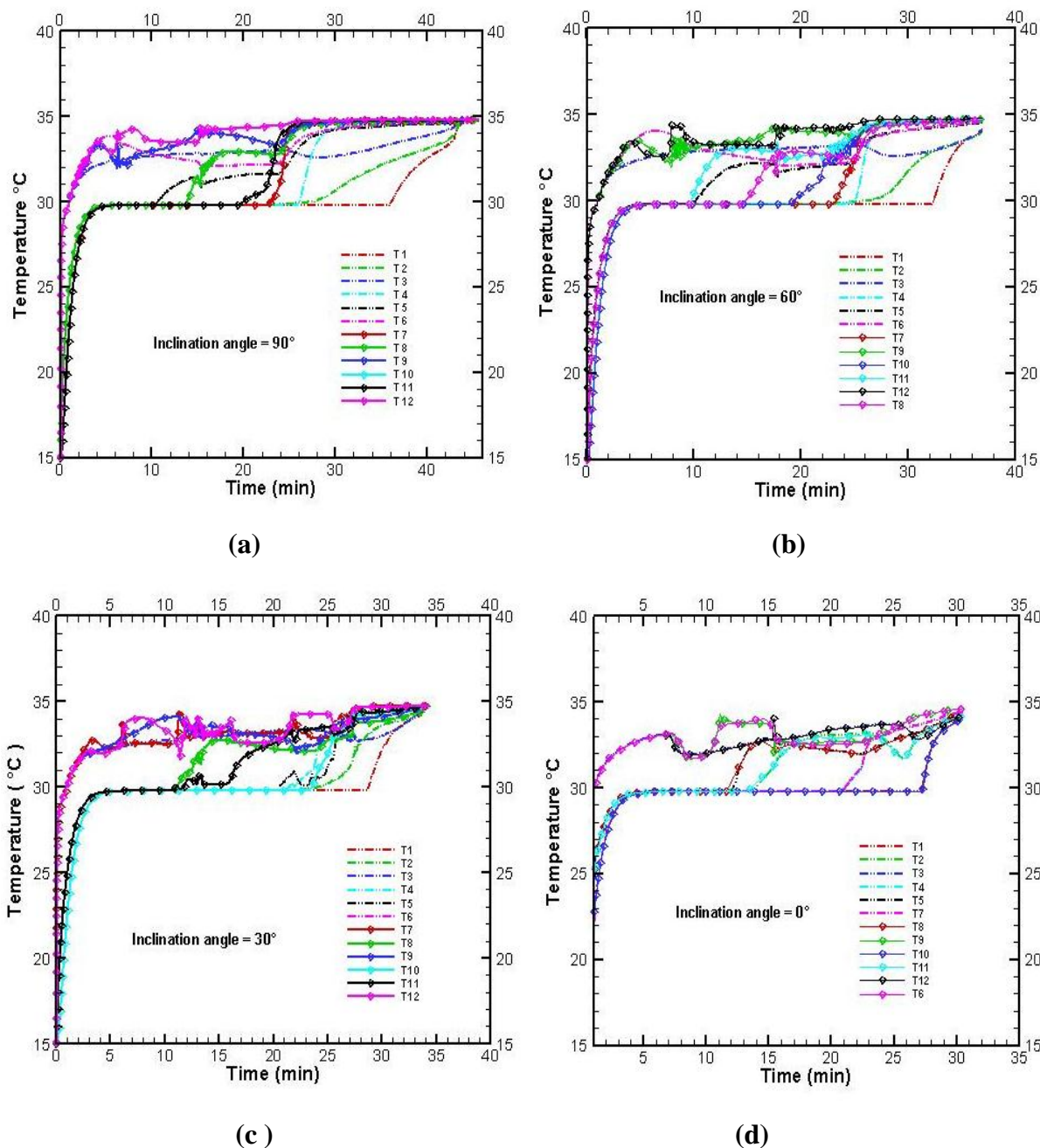


Fig.10: Temperature history at different positions inside the cavity

For all inclination angles, at earlier stages, conduction is the sole mode of heat transfer of the PCM, thus the temperature at different locations increases until the melting temperature is reached. The results show that the rate of temperature increase at positions 3, 6, 9 and 12 is much higher than those at the other ones. An explanation for this is that the heat transfer to the solid PCM at these positions is mainly by heat conduction through the thin layer of the liquid PCM. The decreasing trend of temperature from the upper to the lower positions in the cavity implies the thickening of the thermal boundary layer along the interface

liquid-solid and confirms the presence of the counter-clockwise rotating flow in the liquid. We can also note some minor fluctuations in temperature at positions 9 and 12 for inclination angles 30° and 60° (see Figs. 10-b and 10-c). These fluctuations can be attributed to the three dimensional and unstable flow structures in the liquid PCM. This fact were also observed and reported by Webb and Viskanta [11] and by Kamkari et al. [15].

During the melting process of PCM from below (see Fig.10-d), temperatures at positions 3, 6, 9 and 12 increase uniformly, with no significant fluctuations. We note that the temperatures at positions 2, 5, 8 and 11 show minor fluctuations immediately after the melt front touch them. These fluctuations can be attributed to the regular convection cells in the liquid region and development of turbulent convection currents in the melt layer.

In phase change material melting problems, the rate of melting is a very important parameter. The total liquid fraction in the cavity (ratio of volume of PCM melted to volume of the cavity) is widely used parameter in previous publications. Fig.11 displays the evolution of the total fraction of the liquid in the cavity for different inclination angles.

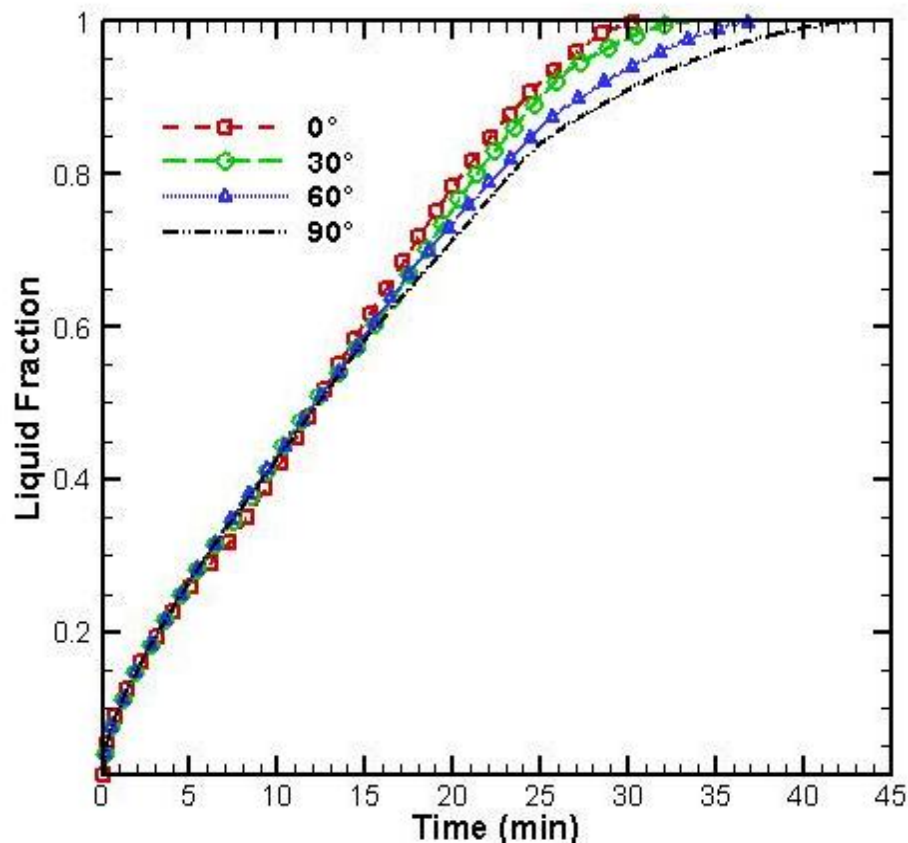


Fig.11: Comparison of the liquid fraction variations versus time for different inclination angles

It appears from this figure that when the convection takes over conduction, the variation of the inclination angle doesn't seem to have influence on the liquid fraction. This is caused by the overwhelming effect of the natural convection regime which suppresses the effect of the inclination angle during the melting process. In the particular case of the horizontal cavity, the liquid fraction vary linearly with time until the end of the melting process, otherwise for the inclination angles 30° , 60° and 90° , the liquid fraction variation deviates from a linear trend and the melting rate decreases by increasing the inclination angle. This is attributed to the suppression of the convection current inside the cavity. This result confirm that in the horizontal position (inclination angle = 0°), the heat transfer rate of a low Prandtl number PCM is not affected by the melt layer thickness during the melting process. Similar results are obtained by Kamkari et al. [15] for high Prandtl number PCM.

c. Conclusion:

Melting process inside a rectangular cavity for different inclination angles has been studied numerically. A decrease in the inclination angle produces irregular liquid-solid interface and increases the strength of the vertical flow structures in the liquid region. The shapes of the liquid-solid interfaces during the melting process in the horizontal cavity show the generation of Railegh Bénard cells in the liquid region. It is also found that the rate of the melting increases by decreasing the inclination angle.

4- Melting of PCM over a vertical wavy surface

a. Introduction

One of the most unfavorable behaviors of phase change materials (PCMs) is the inherent low thermal conductivities which lead to low heat transfer rates into and out of the PCMs. Different strategies have been taken in the past to ameliorate the heat transfer rate in such systems. The present section aims to investigate the amplitude and the number of wavy surface effects on the rate of melting of a square wavy cavity. Roughened surfaces are more efficient in heat transfer because of their capability to promote fluid motion near the surface.

A lot of review works are devoted to discuss various PCM types along with specific applications, thermo physical properties, heat transfer enhancement and system-related issues. The main objective of these studies is to enhance the heat transfer and melting rate inside different types of encapsulation [35-39]. Gong et al. [40] has highlighted the effect of inverting a square PCM container heated from the side when thermal stratification in the melt slows the melting rate during the melting process, Gong[40] used the streamline Upwind/Petrov–Galerkin finite element method combined with a fixed-grid primitive variable method. The effect of adding nanoparticles to the PCM in the same cavity was studied numerically by Khodadadi and Hosseinizadeh [37], They found that the freezing rate increases when the cavity is cooled laterally, because of the presence of nanoparticles. In [41], Fteiti and Ben Nasrallah studied the melting process of PCM in a square enclosure heated from below; they found that faster melting is exhibited in flat enclosure. The same qualitative results were found by Hernandez-Guerrero et al. [42]. In [35], El Omari et al used different geometries filled with thermal conductivity-enhanced phase change material in a passive cooling system.

The present section aims to study the melting process in a wavy rectangular cavity; the wavy surface is subjected to a relatively higher temperature than the melting temperature of the phase change material (PCM). The impact of the wavy surface on the natural convection flow coupled to the melting process is presented. The enclosures studied are filled by the same volume of PCM.

b. **Results and discussion:**

Fig.12 shows the computational meshes used to study the effect of the wavy surface on the kinetics melting of the PCM inside the enclosure.

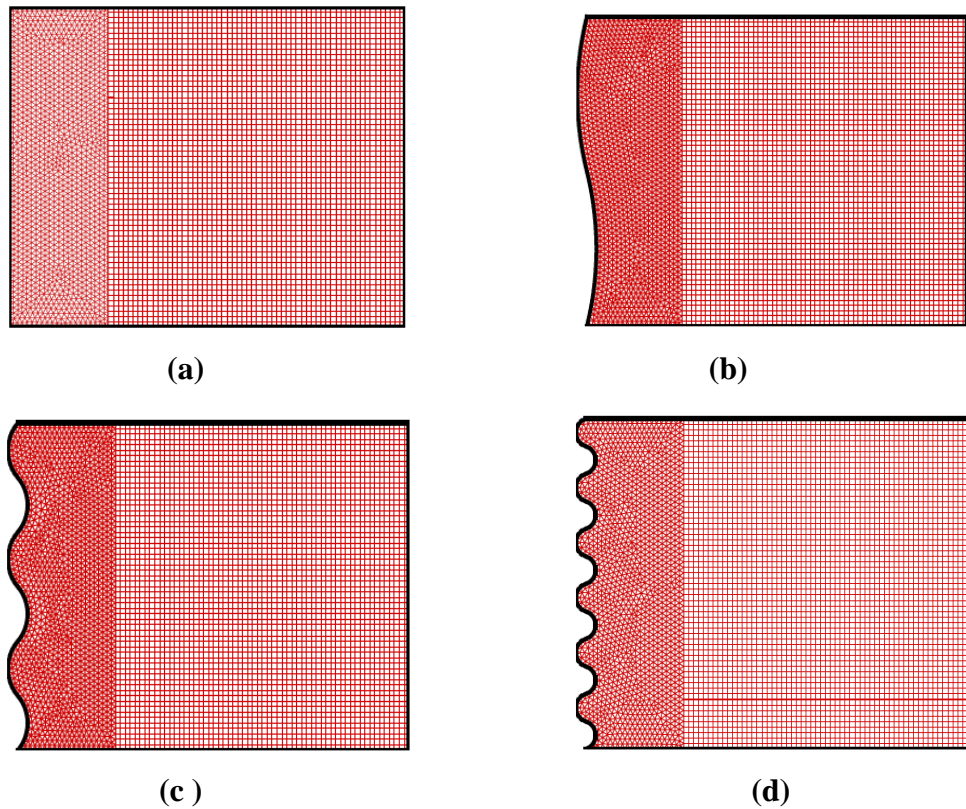


Fig.12: The geometries of the four studied enclosures: a, b, c and d and corresponding meshes.

In these configurations two geometric parameters are explored for the characterization of the temperature and flow fields in the rectangular cavity during the melting process: the number of undulation b and the amplitude a , of the vertical wavy surface. In the present numerical investigation, the dimensions of the cavity used are: $L = 6$ cm and $W = 8$ cm the PCM used is solid Gallium initially at temperature T_C . The horizontal walls are supposed to be insulated. The left hot wall and the right cold wall are maintained at temperatures, $T_H = 38.3^\circ\text{C}$ and $T_C = 28.3^\circ\text{C}$, respectively. a and b are chosen to be $0 \text{ mm} \leq a \leq 6 \text{ mm}$ and $0 \leq b \leq 6$.

Figs.13 and 14 show the effect of the number of undulations on the streamlines and isotherms for several times.

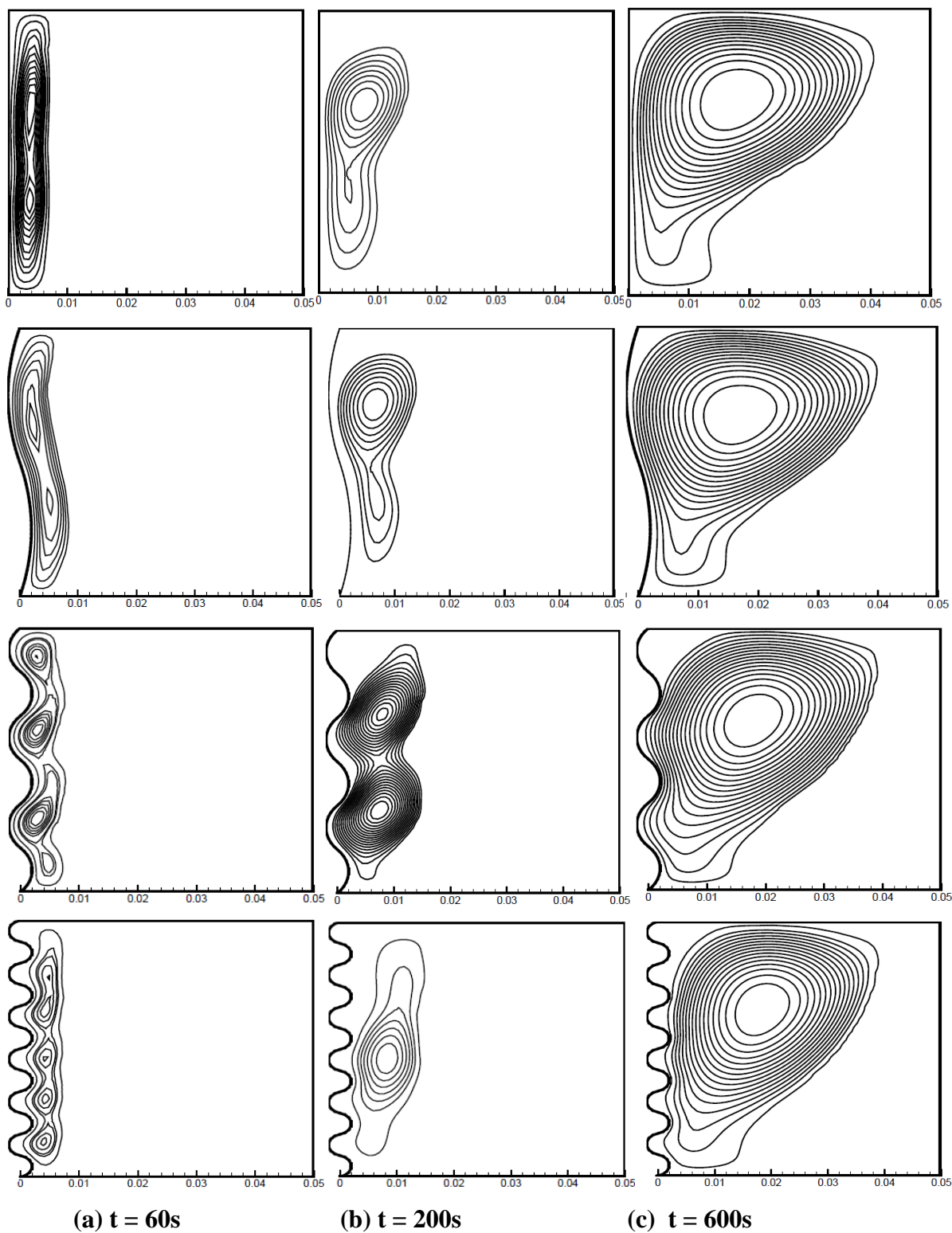


Fig. 13: Streamlines at times 60s, 200s and 600s for various undulations ($a = 2mm$)

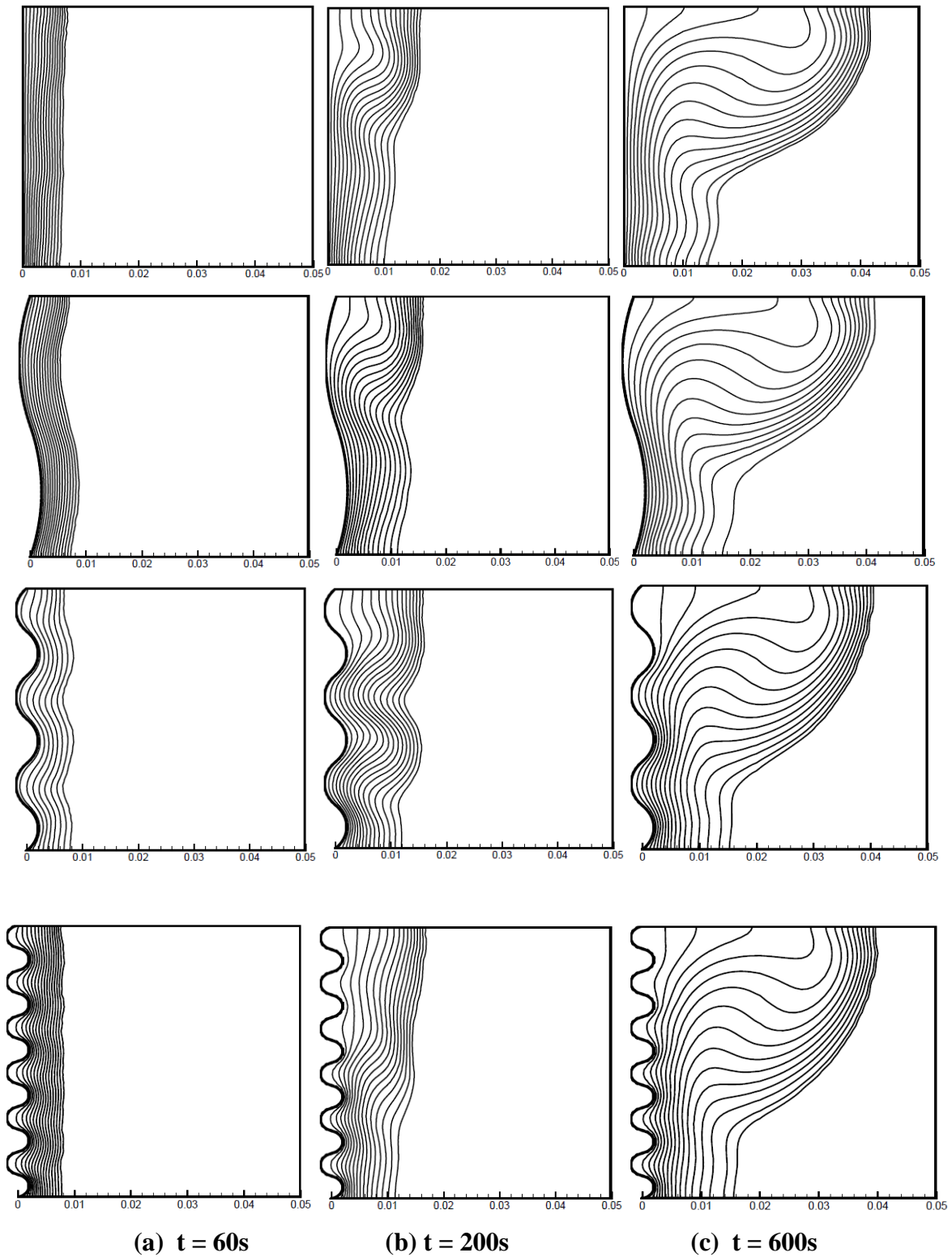


Fig. 14: isotherms and liquid-solid interfaces at times 60s, 200s and 600s
for various undulations

The left boundary of each picture corresponds to the left hot wall and the amplitude value of the wavy surface is maintained at $a = 2$ mm.

These results show clearly that the number of undulations disturb the heat transfer near the hot wall. At the first stages, conduction predominate the heat transfer in the melt zone, and the liquid-solid interface takes the form of the heated wall. As melting progresses, convection takes over conduction and two convective Bernard cells appears after 60 s, the shape of the interface has distinct bulges for 6 undulations. After 200 s, recirculating eddy establishes in the cavity; there size increases as melting proceeds. In the upper side of the solid-liquid interface, warm fluid strikes the interface after being heated by the hot wall witch lead to high melting rate there. The temperature of the fluid diminishes relatively as the fluid descends along the interface transferring the heat to the melting front. Consequently, the melting rate at the top is significantly higher than that at the bottom. We note that as the melting progresses, the effect of the surface waviness on the liquid-solid interface diminishes. The manner of how convection influences overall conduction in the cavity is gauged by the help of the local Nusselt number at the hot wall. The effect of the number of undulations on the Nusselt number for several times is presented in Fig.15. Sudden drops of the Nusselt plot appear clearly at times corresponding to roll pairings. They are more pronounced for the wavy surface with 12 undulations. We note the apparition of a number of peaks and valleys corresponding to the imposed undulations, their magnitude decreases as melting progresses.

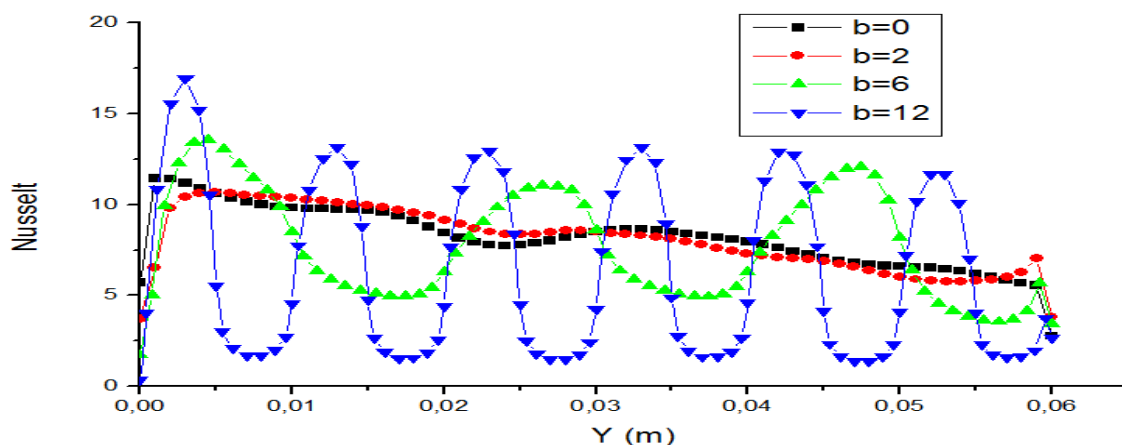
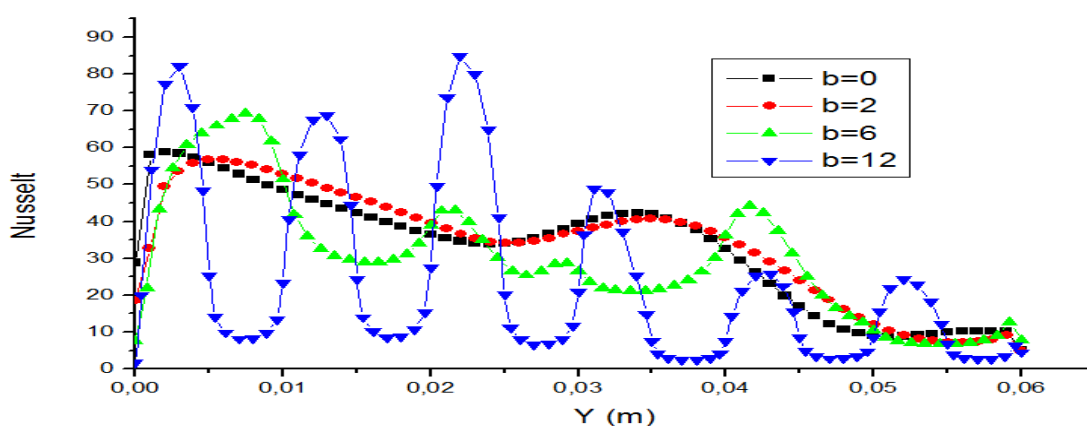
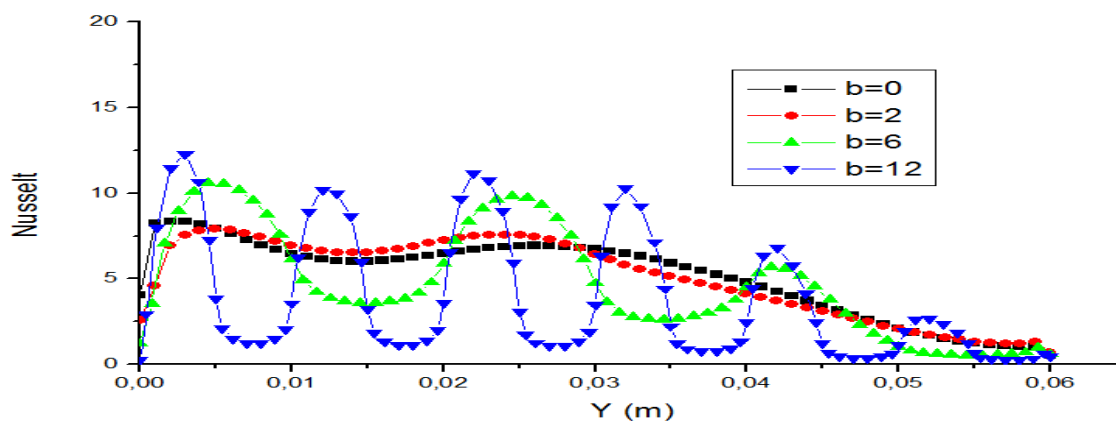
(a) $t = 60$ s(b) $t = 200$ s(c) $t = 600$ s

Fig. 15: Nusselt number at times 60s, 200s and 600s for various undulations.

Another important parameter that characterizes the melting process is the melting rate. The time evolution of the ratio volume of the melt to the total volume of the cavity (total liquid fraction in the cavity) is a factor that has been widely used as a monitoring parameter in

previous publications. The slope of the tangent line of the liquid fraction versus time plot at a given time represents the melting rate whereas the average melting rate is the ratio of current liquid fraction and time.

The total liquid fraction in the cavity versus time plot is presented in Fig.16. It appears clearly that when melting progresses there is no effect of varying the number of undulations on the evolution of liquid fraction versus time.

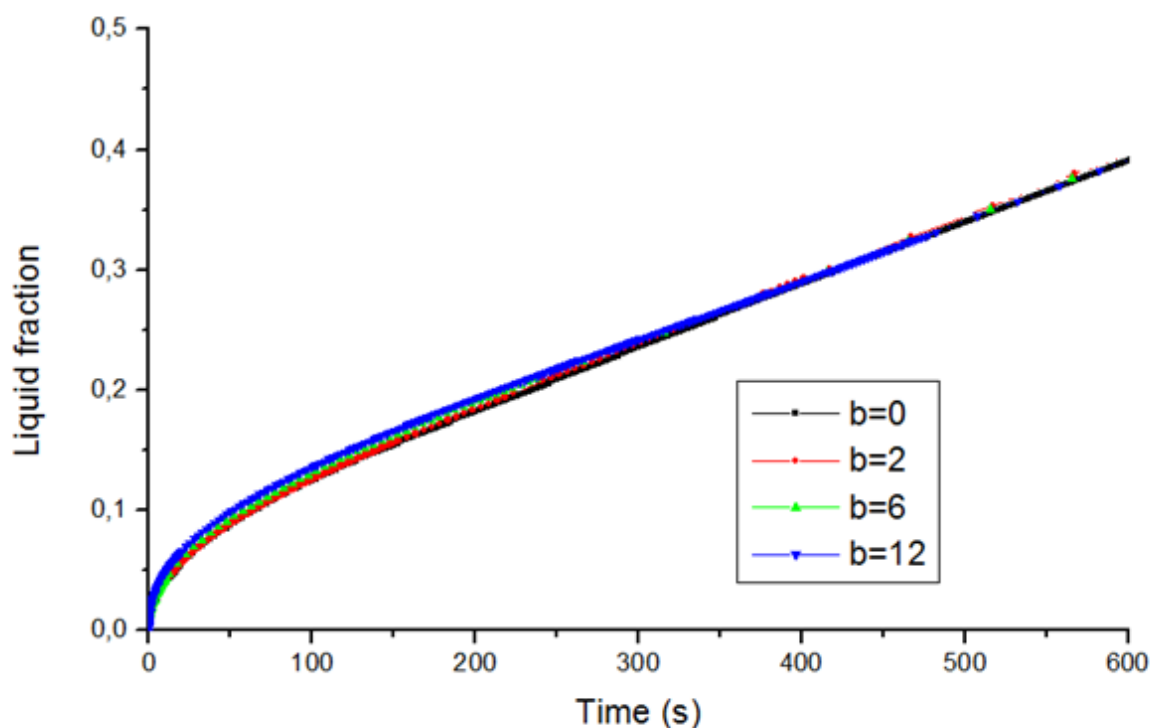


Fig. 16: Liquid fraction versus time for various undulations.

The effect of varying the amplitude “a” of the wavy surface on the streamlines and temperature contours is also demonstrated in Figs. 17 and 18 for several times and for two undulations.

The variation of the wavy surface amplitude between 0 mm and 6 mm disturbs the general flow and isotherm patterns in the vicinity of the wavy surface. This effect is clearly seen for $a = 4$ mm and $a = 6$ mm at $t = 200$ s where distinct bulges appear on the shape of the interface due to the presence of two convective rolls. However, as melting progresses, the convection takes over conduction and the effect of the amplitude of the wavy surface on the liquid-solid interface diminishes.

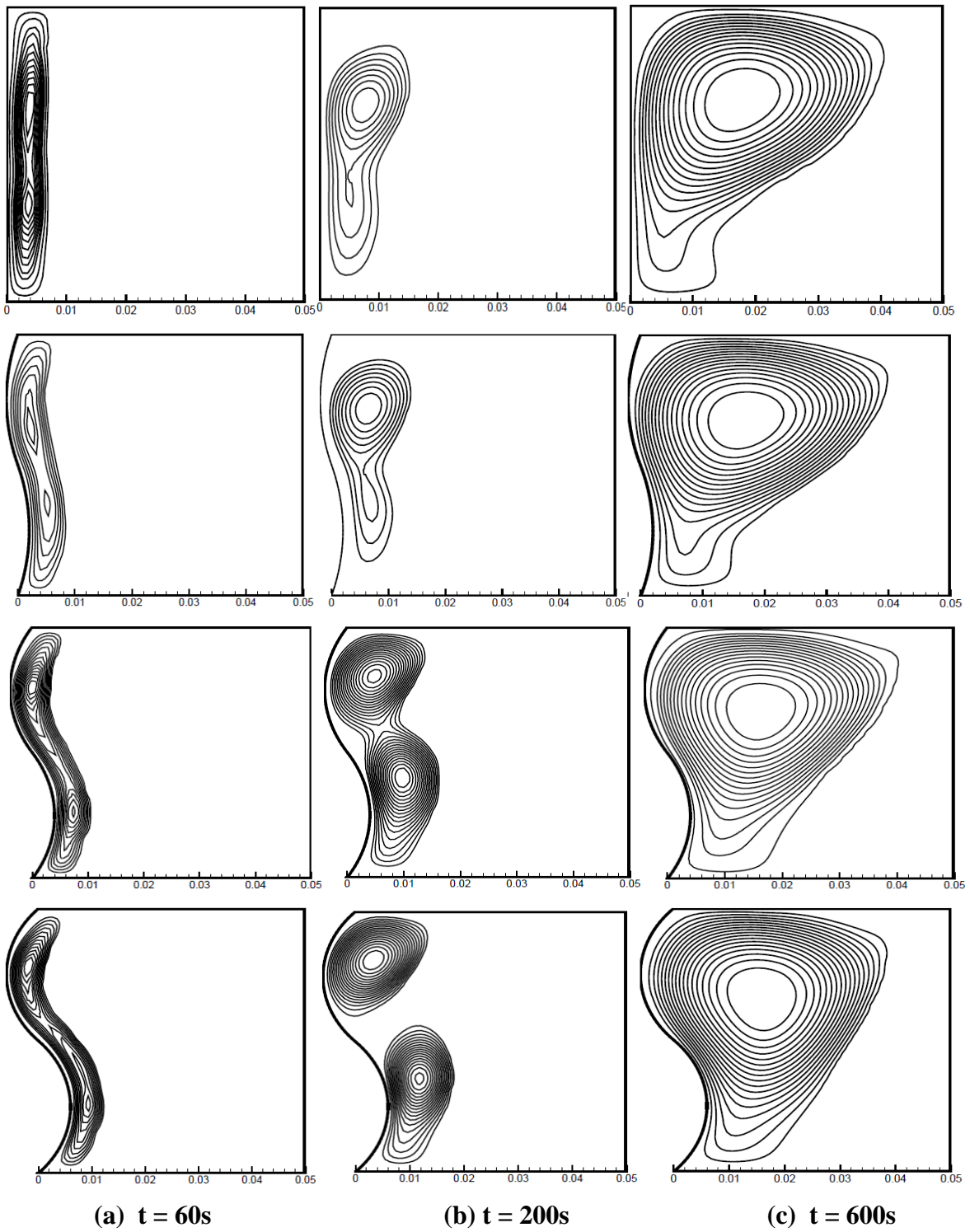


Fig. 17: Streamlines at times 60s, 200s and 600s for various amplitudes

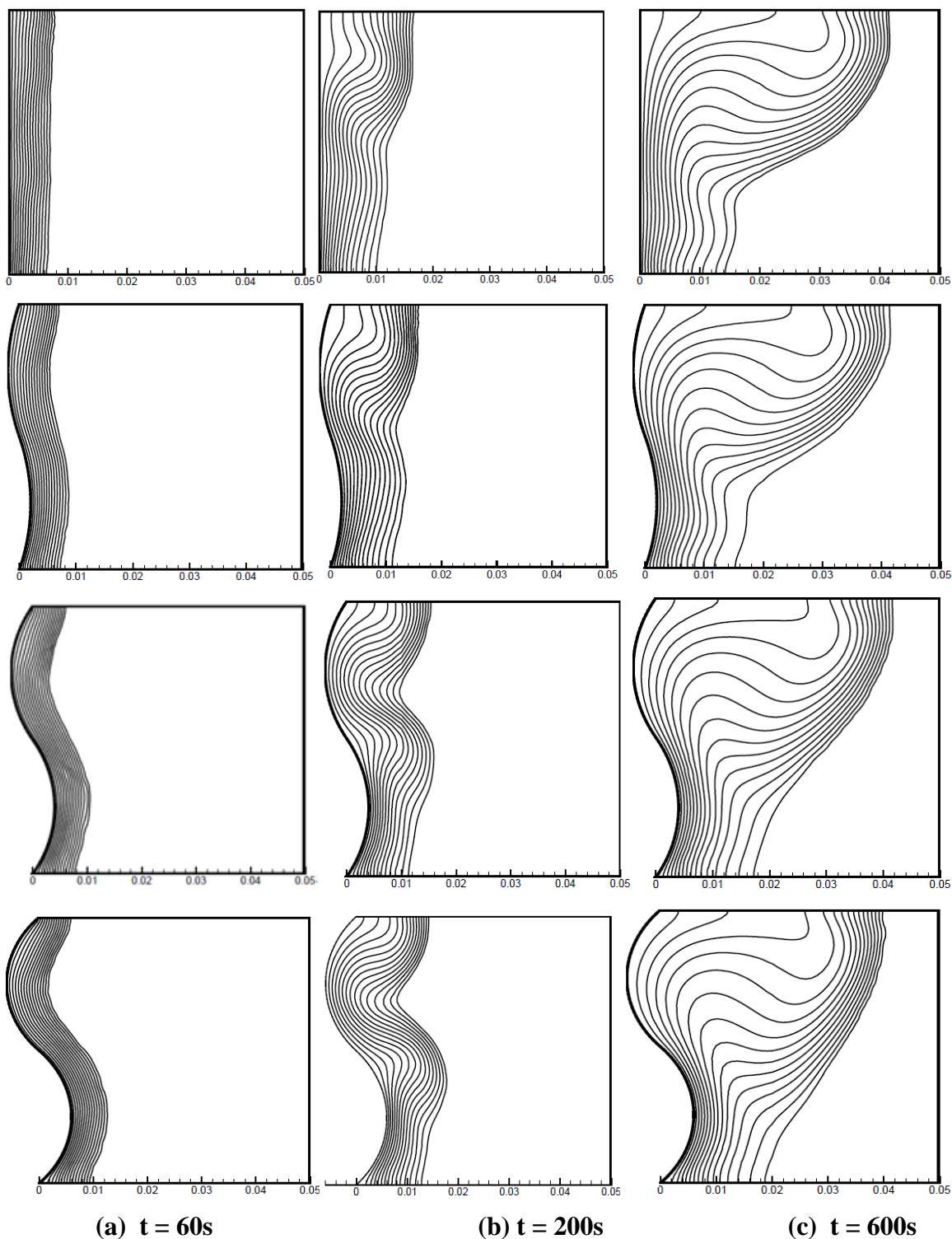


Fig. 18: Temperatures contours and interfaces liquid-solid at times 60s, 200s and 600s for various amplitudes

The effect of the variation of the wave amplitudes on the Nusselt number for several times is shown in Fig.19: We note that at $t = 200$ s for $a = 4$ mm and $a = 6$ mm sudden drops

appear in the Nusselt plot, this is due the presence of two convective rolls in the flow structure. As melting progresses the magnitude of the Nusselt number decreases.

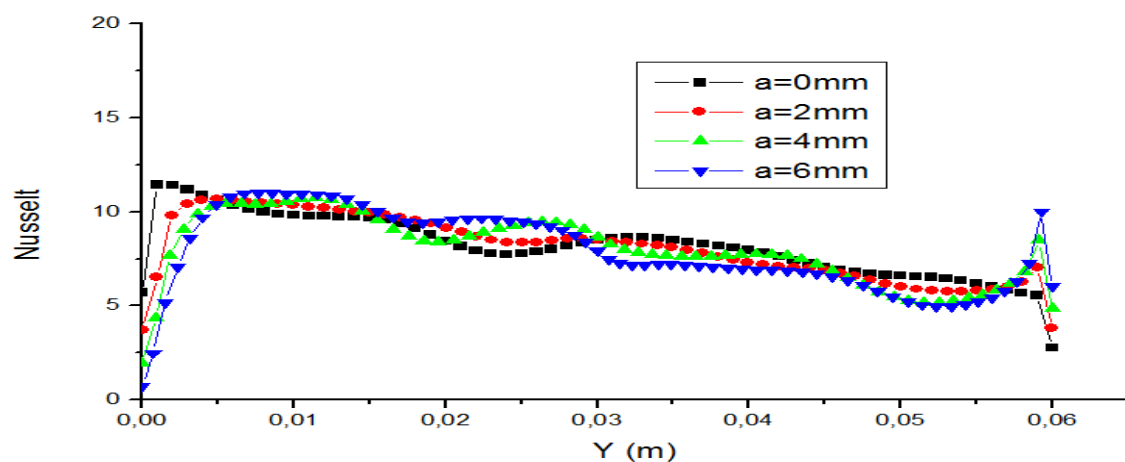
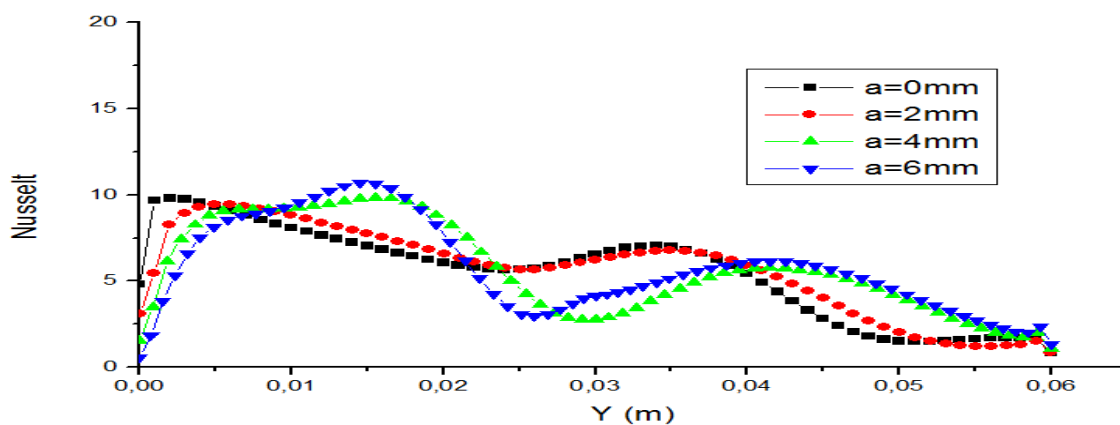
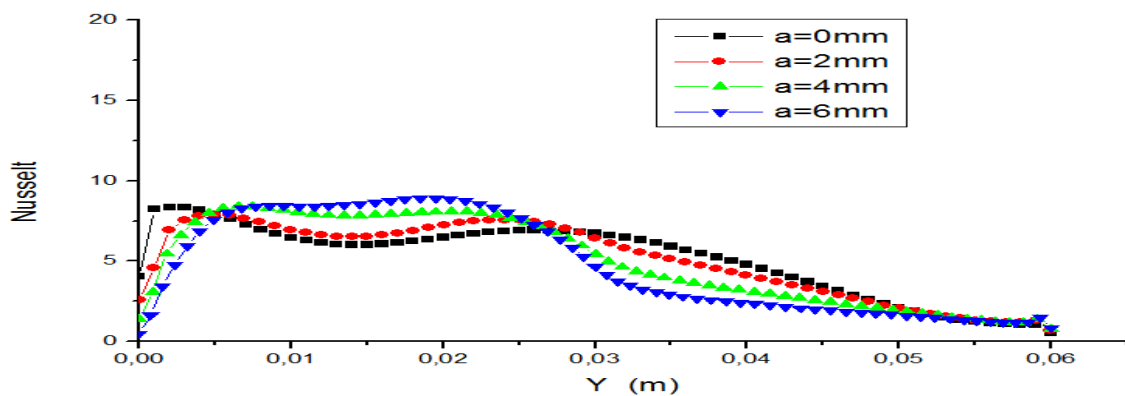
(a) $t = 60$ s(b) $t = 200$ s(c) $t = 600$ s

Fig. 19: Nusselt number at times 60s, 200s and 600s for various amplitudes

Fig.20 represents the time evolution of the total liquid fraction for different amplitudes.

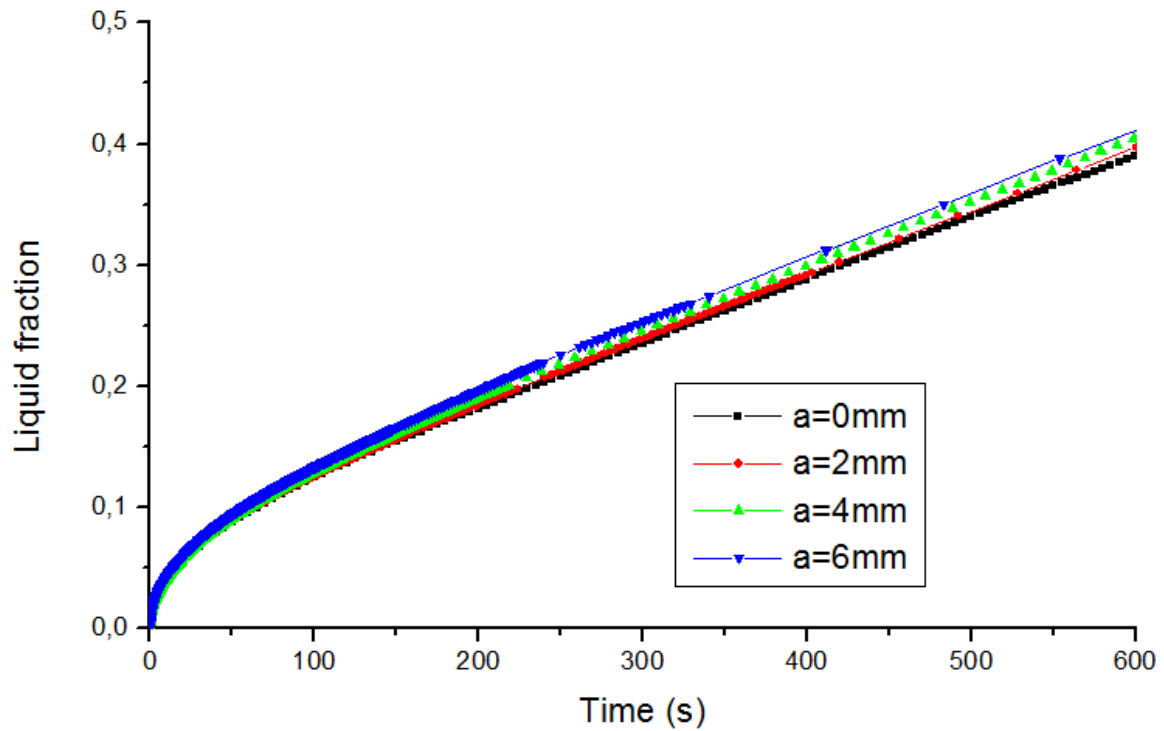


Fig. 20: Time evolution of the liquid fraction for various amplitudes.

At early times, when conduction is still the predominating heat transfer mode in the melt zone, the magnitude of the wavy surface amplitude doesn't seem to have an effect on the evolution of liquid fraction. However, as time progresses, the convection becomes the predominant heat transfer mechanism, and the rate of melting increases with the magnitude of the amplitude value of the wavy surface.

5- Melting over a horizontal wavy surface

a. Introduction:

Experimental and numerical studies for convection dominated melting of PCMs have concerned different geometry encapsulation, but little effort was devoted to the melting of a PCM heated from below on a rectangular wavy surface cavity [40, 41, 46-51]. The excitation of the wavy surface creates a cellular form of natural convection inside the fluid zone, as in a classical Rayleigh-Benard convection.

Experimental investigations were performed by Yen and Galea [51, 52] and Seki et al. [53] on the melting process of a horizontal ice slab excited from below. Other experimental works were done by Hale and Viskanta [83], they studied in a rectangular cavity, melting from below and solidification from top of n-octadecane, they did not present the liquid solid interface shapes and the flow patterns. Gau et al. [55] presented the visualization of the flow for melting from below of an n-octadecane slab in a rectangular cavity. As an extension of Gau et al experiments, Diaz and Viskanta [56] presented morphology observation of the phase change interface.

Numerical studies on the melting of a pure substance within a vertical cylindrical enclosure heated isothermally from below was performed by Prud'Homme et al. [48], the vertical side was assumed to be adiabatic. Computer generated body-fitted curvilinear coordinates were used to solve for the governing equations for the convective flow in the melt. The main conclusion is that the critical Rayleigh number for the onset of convection based on the melt layer thickness is 2197. Lacroix [46] used a variable grid method for the melting of a low Prandtl PCM, he performed numerical simulations by analyzing the effect of the temperature difference between the bottom wall and the initial temperature of the PCM. Gong and Mujumdar [47] investigated numerically the effect of the Rayleigh number on the flow patterns during the melting of a pure substance in square cavity heated from below. More recently, Fteiti and Nasrallah [41] study the melting process in rectangular cavity heated from below and cooled from above, the control volume based finite element method was used to solve the governing equations and the enthalpy-porosity technique to track the phase change interface motion. They concluded that smaller is the aspect ratio, more rapid is the melting process, but at the steady state the fraction of the volume melted is less important.

Our objective in the present section is to study the process of melting of the gallium in a rectangular cavity by exciting the bottom wavy surface by a relatively higher temperature

than the melting temperature of the material. The effect of the wavy surface amplitude on the melting of the PCM is investigated.

b. Result and discussion

The structure concerned by the current numerical investigation is a square rectangular cavity (6 cm × 6 cm) containing solid PCM Gallium initially at $T_i = 28.3^\circ\text{C}$. The wavy surface is kept at temperature $T_B = 38.3^\circ\text{C}$. The other walls are considered to be adiabatic. The wavy surface amplitude varies between 0 mm and 14 mm. The thermo-physical property values were taken at 32°C , which correspond to the experiment temperature range. The non-dimensional numbers at this temperature are: Prandtl number: $Pr = 0.0216$, the Stefan number: $St = 0.039$ and the Rayleigh number: $Re = 10^5$.

Figs.21, 22, 23, 24 and 25 illustrate how the wavy surface amplitude affects the streamlines and temperature contours for several times. The undulation number is fixed at $b = 2$. The results show clearly that a change in the amplitude of the wavy surface disturbs the heat transfer in the proximity of the heated wall. At first stages, heat transfer by conduction is the predominating mode in the melt zone; the liquid-solid interface takes the form of the wavy surface. However, when melting progress, convection takes over conduction. After the conduction dominating stage a complex structure appears in the liquid zone, characterized by a multi-cellular flow patterns, appears. The number and the size of the Benard cells are dependent on the height of the liquid fluid. Four counter-rotating cells appear at $t = 400$ s for $a = 0$ mm, and three counter-rotating cells are present for $a = 4, 8, 14$ mm as shown in Fig.23.

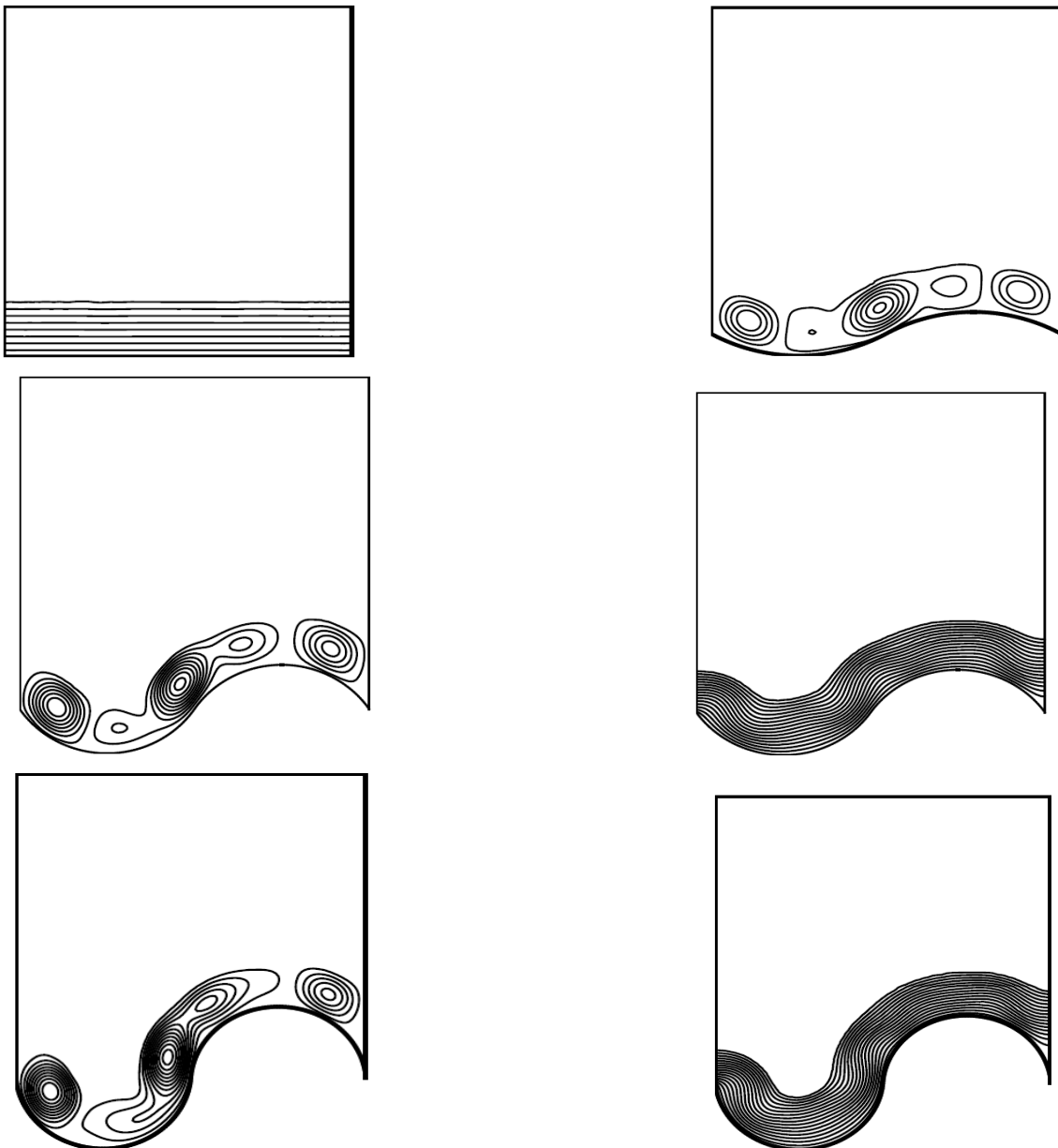


Fig.21: Streamlines and isotherms at time 100s for various amplitudes.

The three-dimensional convection is neglected in the present study, since we use a two dimensional model, and the three dimensional convection happens very shortly compared to the entire melting process.

The rolls at the center disappear as the liquid layer increases whereas those at the bottom corners decrease. At approximately 50% of the melting process ($t = 600$ s), two identical cells appears for $0 \text{ mm} < a < 14 \text{ mm}$.

An important result in this investigation is that the interface shows a trough in the center, a possible explanation for this is that the hot fluid rises through the vertical surfaces

and falls from the center of the cavity. The same rolls number is mentioned in some other numerical works, as in [76], but the circulation of the fluid is different to our results. The flow becomes unsymmetrical and one of the two cells becomes arbitrarily dominant. The front advances more rapidly near the dominating cell.

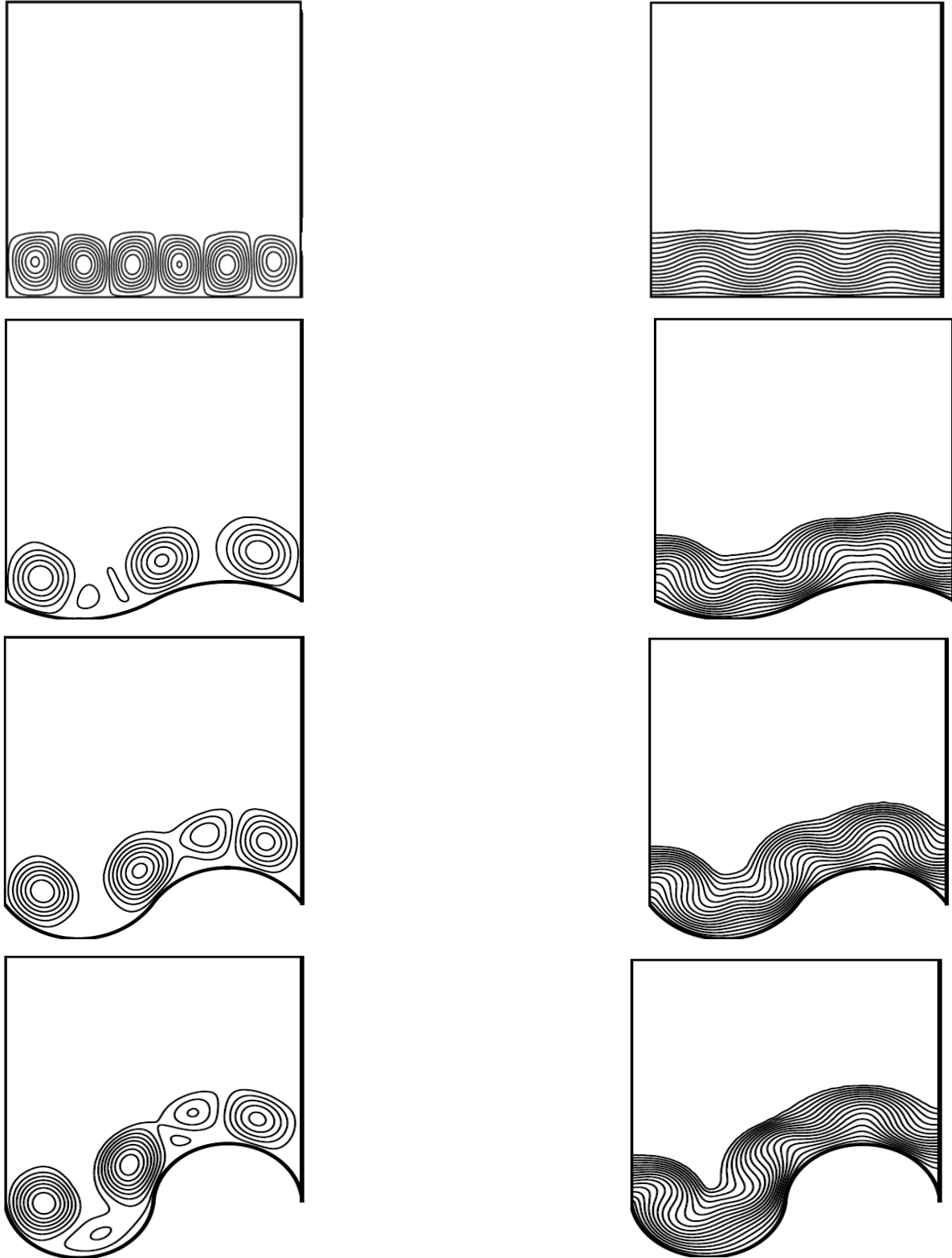


Fig.22: Streamlines and isotherms at time 200s for different amplitudes.

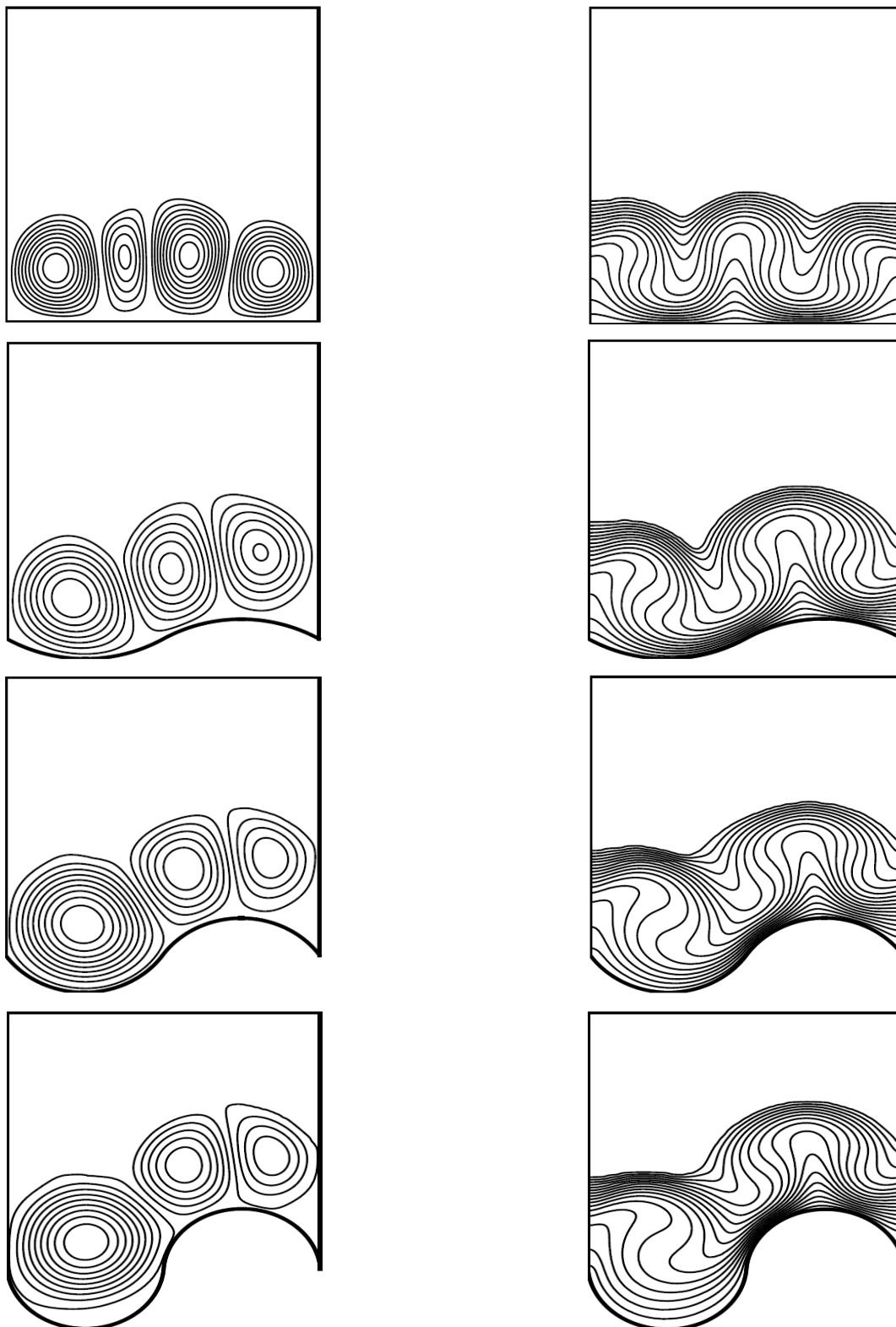


Fig.23: Streamlines and isotherms at time 400s for different amplitudes.

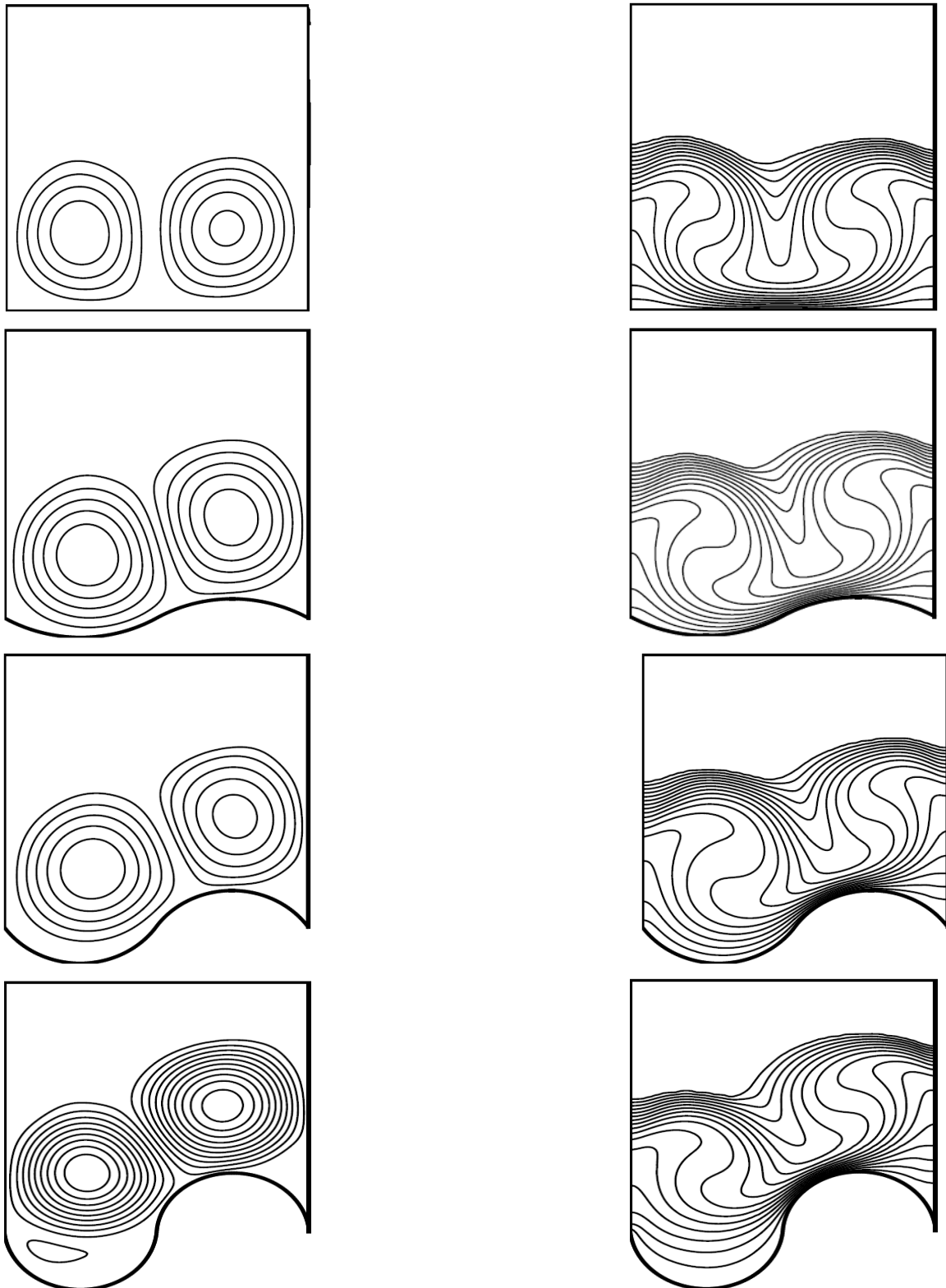


Fig.24: Streamlines and isotherms at time 600s for different amplitudes.

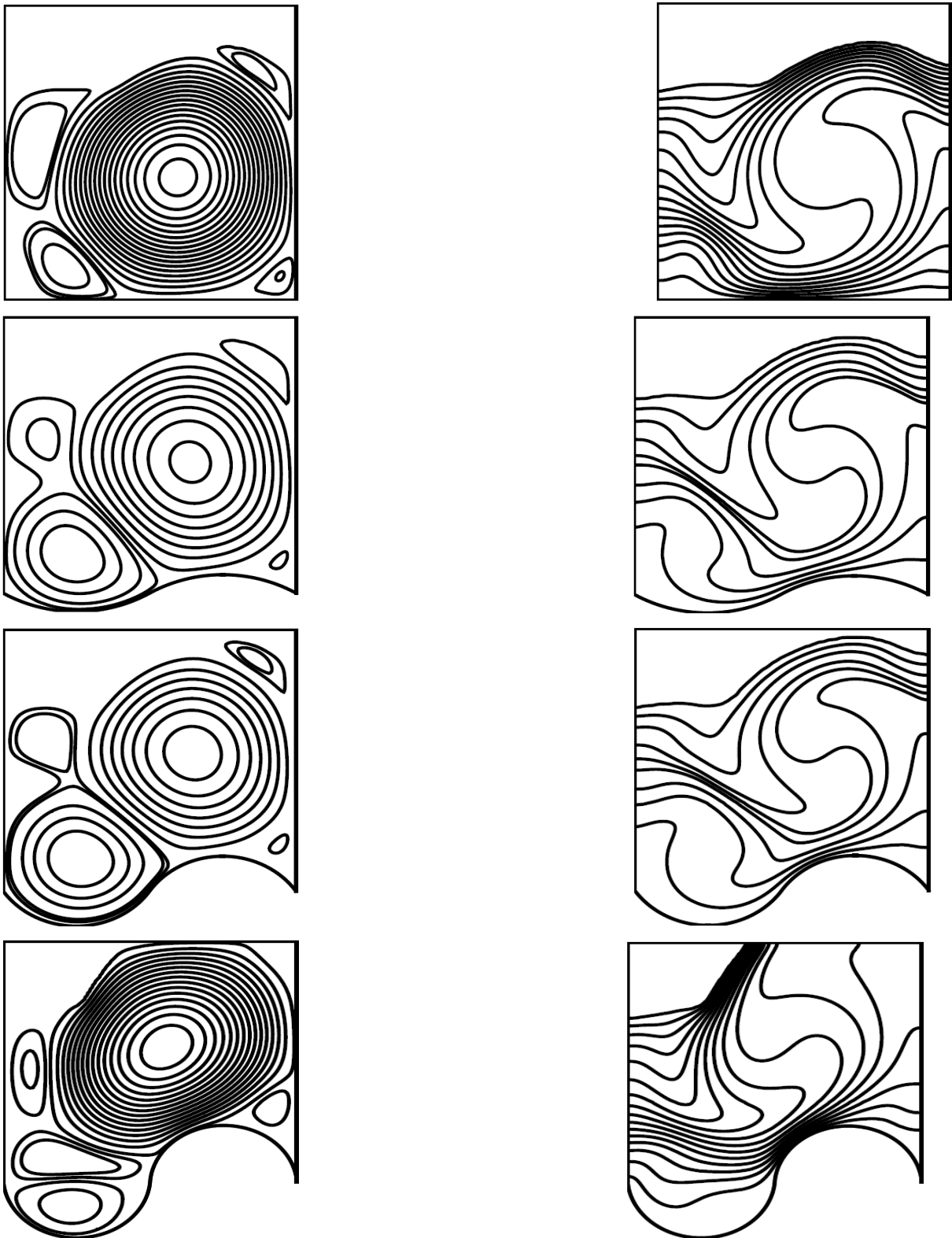


Fig.25: Streamlines and isotherms at time 1000s for different amplitudes.

In order to show the effect of the convection over the global conduction through the encapsulation, the local Nusselt number at the hot wall is calculated for different amplitudes and for several times. The Results are shown in Figs.26 and 27.

Sudden variations of the Nusselt plot appear at times relative to roll pairings. It is clearly seen that the amplitude of the nodulation influence the distribution of the local heat flux near the wavy surface. As melting progresses the magnitude of the peaks increases.

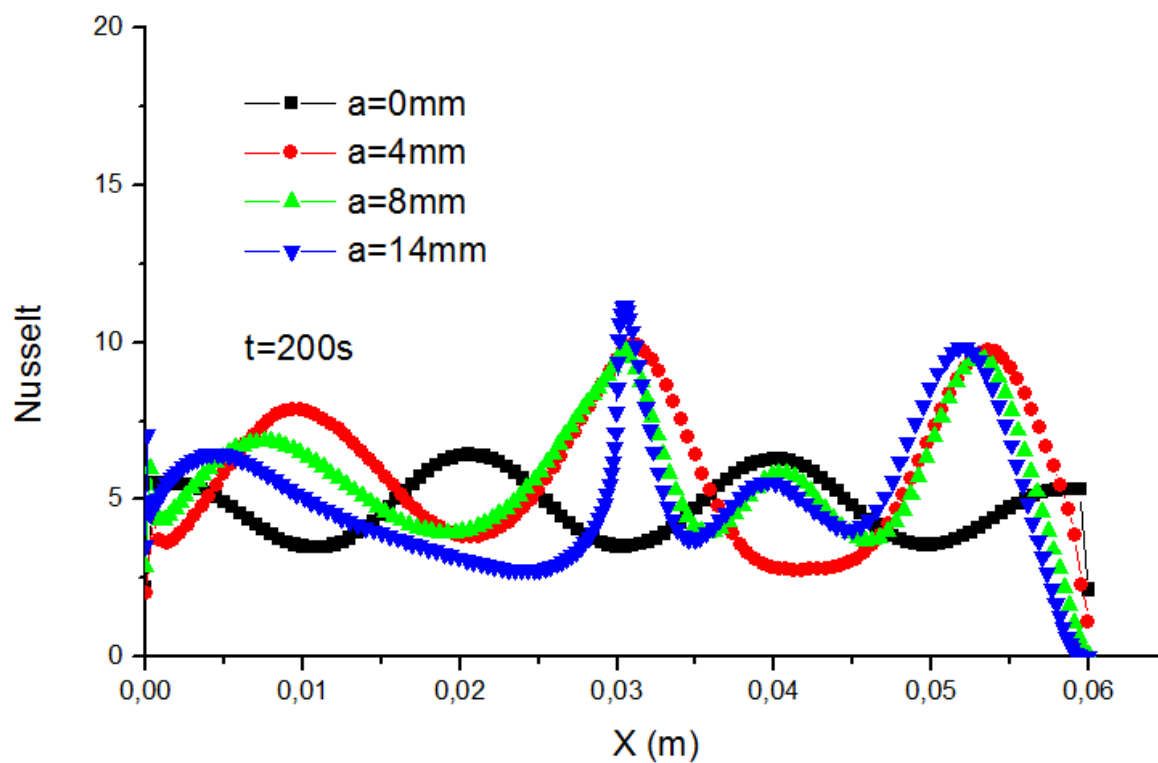


Fig.26: Nusselt number at time 200s for different amplitudes.

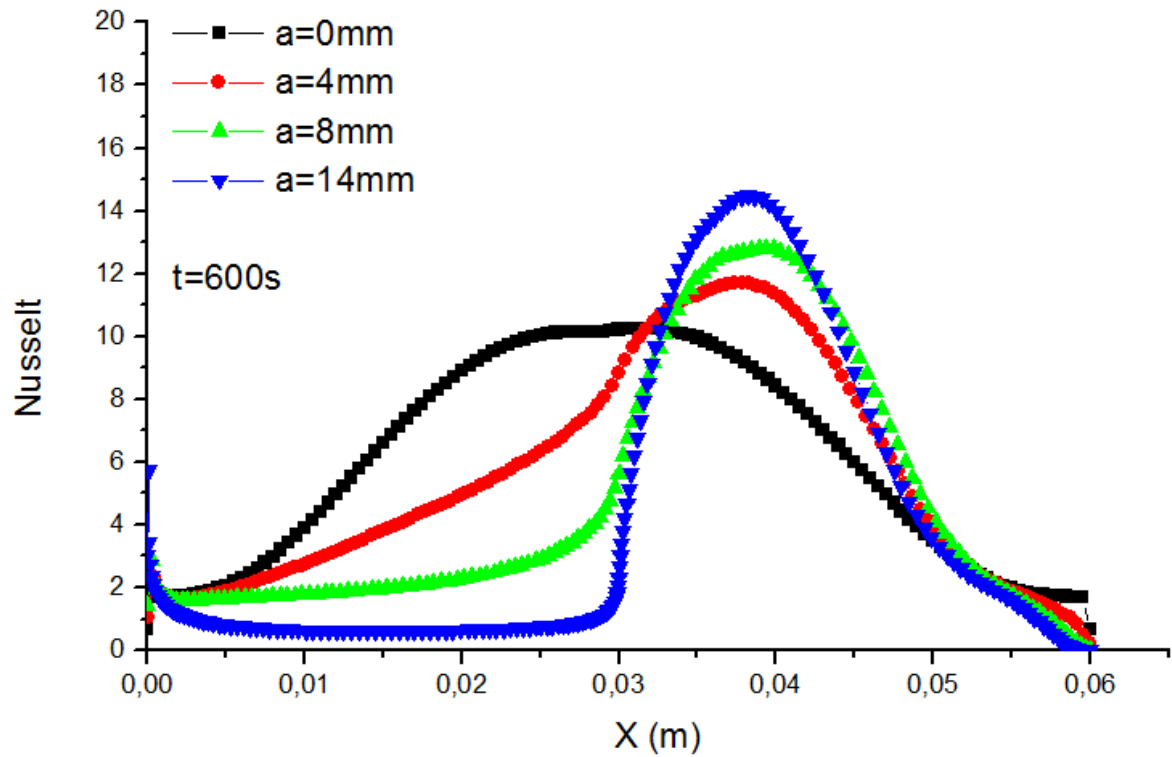


Fig.27: Nusselt number at time 600s for different amplitudes.

The evolution of the total liquid fraction in the cavity is presented in Fig.28. The main conclusion is that an increase of the nodulation amplitude results in rapid melting inside the cavity.

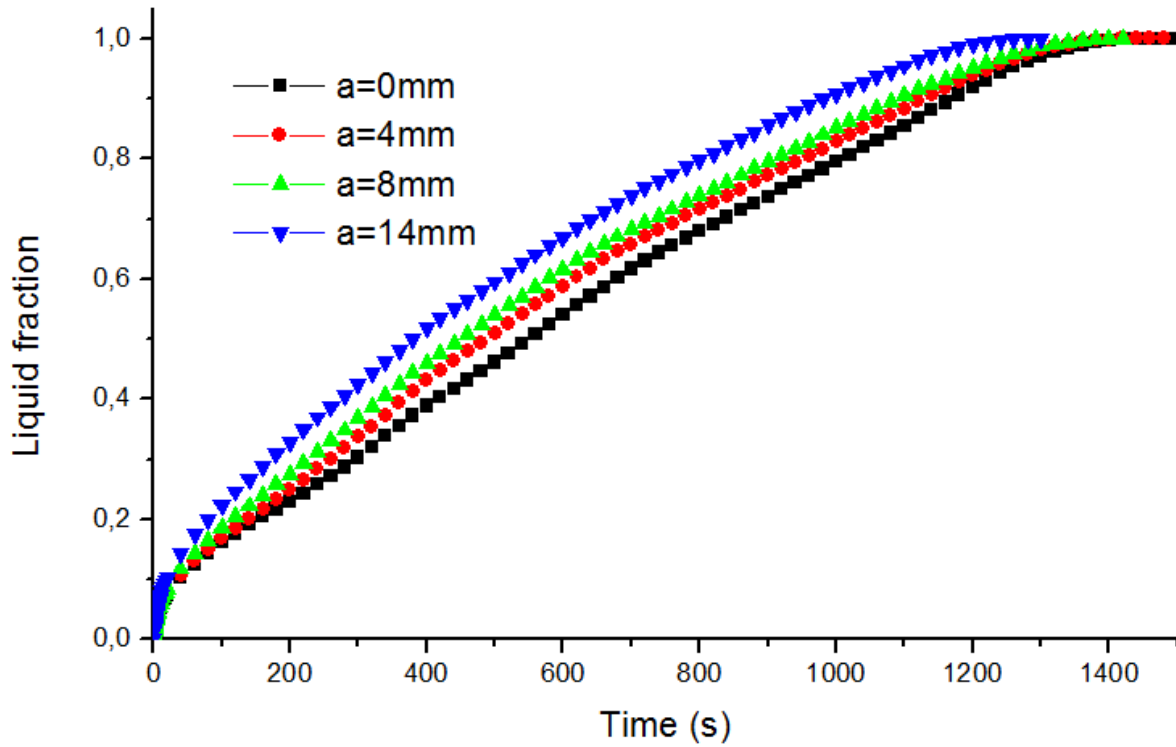


Fig.28: variation of liquid fraction for different amplitudes.

6- Conclusion

In this section, we have investigated numerically the melting process on a rectangular cavity heated from a vertical then a horizontal, undulated surface with uniform temperature. Increasing the amplitude magnitude of the wavy surface ameliorates the rate of the melting of the PCM. As continuation of the present work we are investigating how to integrate efficiently such a device into a real system.

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Chapter III:

PCM Melting in Cylindrical Enclosure

1- PCM Melting in a vertical cylinder

a. Introduction

Latent energy storage (LES) is required to ensure the continuity of a thermal process in energy systems where a temporal difference exists between the supply of energy and its utilization [1-3]. Certainly, LES is of particular interest and significance in using this essential technique for solar thermal applications such as heating, hot water, cooling, air-conditioning, etc., because of its intermittent nature. In these applications, a LES system must be able to retain the energy absorbed for at least a few days in order to supply the energy needed on cloudy days when the energy input is low. Good understanding of heat transfer during melting process is essential for predicting the storage system performance with accuracy and avoiding costly system overdesign [4-5]. Natural convection heat transfer around and within cylindrical capsules finds various practical applications in space heating, heat exchangers, solar energy collectors, energy storage systems, and electronic devices. During the solidification process, conduction is the sole transport mechanism but in the case of melting natural convection occurs in the melt region and this generally enhances the heat transfer rate compared to the solidification process.

Several authors have tackled the problem of natural convection dominated melting inside a horizontal tube and inside a horizontal cylindrical annulus [6-8]. The melting process was investigated analytically and experimentally for a wide range of thermal conditions and melting scenarios.

Heat transfer during melting inside an isothermal horizontal capsule has been studied by Reiger et al. [9-10]. They tract numerically the flow patterns, the interface positions, and the temperature fields of the PCMs for different wall temperatures of the capsule. In [11] Zivkovic et al. investigated the PCM melting process inside rectangular and cylindrical capsules. Saitoh and Hirose[12] and Bareiss and Beer[13] studied the effect of the density variation between the solid and the liquid phases on the melting process

Nevertheless, only few studies are devoted to the issue of melting inside vertical cylindrical enclosures, such as studies of Bareiss and Beer[13] and Ei-Dessouky et al.[14]. As results, they found that maximum liquid fraction and PCM temperature occurs at the top of the heater, therefore the highest rate of melting occurs in that region.

Sparrow et al. [15] investigated experiments on the melting of a PCM in a vertical isothermal tube. A pure conduction model was used for comparison proposes. The authors

found that the energy transfer associated with the melting process in the experiments were about 50% higher than those predicted by the conduction model. In [16], Wu and Marcel studied numerically the melting process of PCM inside a vertical cylindrical capsule. Their main conclusion was that, when convection is fully developed in the melt, the heat transfer rate at the top is decreased to zero, and the highest heat transfer rate is observed at the bottom of the cavity

Numerical research of melting inside a vertical cylindrical enclosure, warmed up at a constant temperature from below, was conducted by Prud'homme et al. [17]. They demonstrated that the sharp and the motion of the solid –liquid interface are strongly influenced by the multicellular convective flow patterns at the bottom of the enclosure, the influence was also observed on the Nusselt number at the bottom wall witch exhibited strong local variations.

The main conclusion of these studies is that the natural convection in the liquid is the major heat transfer process along with conduction in the melting process. The goal of the present numerical investigation is to study natural convection dominated melting of a PCM within a vertical cylindrical capsule.

b. Results and discussion

Using the above-described model simulations were carried out for melting of a PCM (Gallium) within a vertical cylinder. Numerical investigations were conducted using 65000 cells and the time step of 10^{-3} s was found to be sufficient to give accurate results. The two configuration cases are presented in Fig. 29:

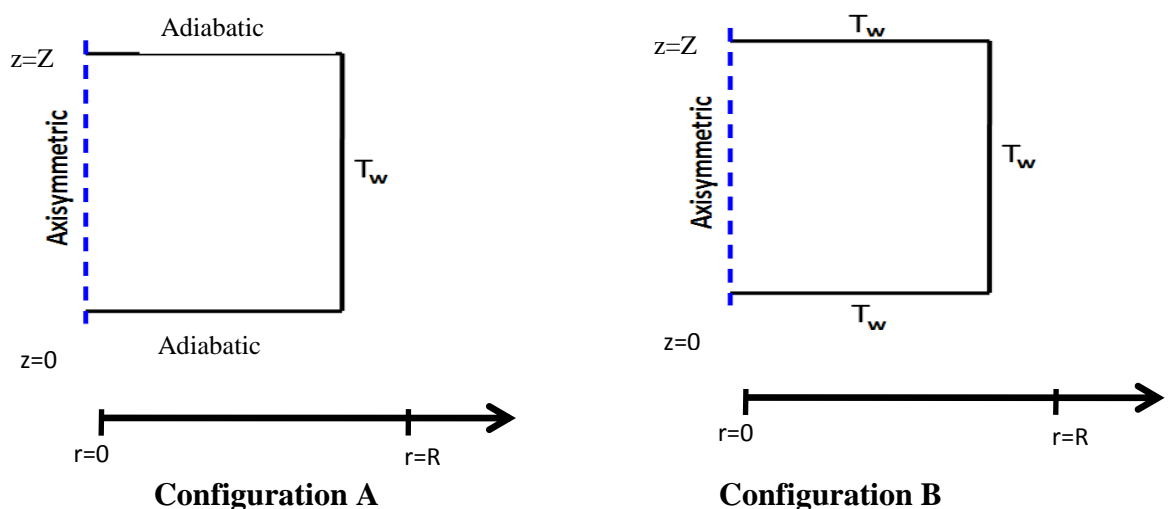


Fig.29: The schematic of the two configurations studied.

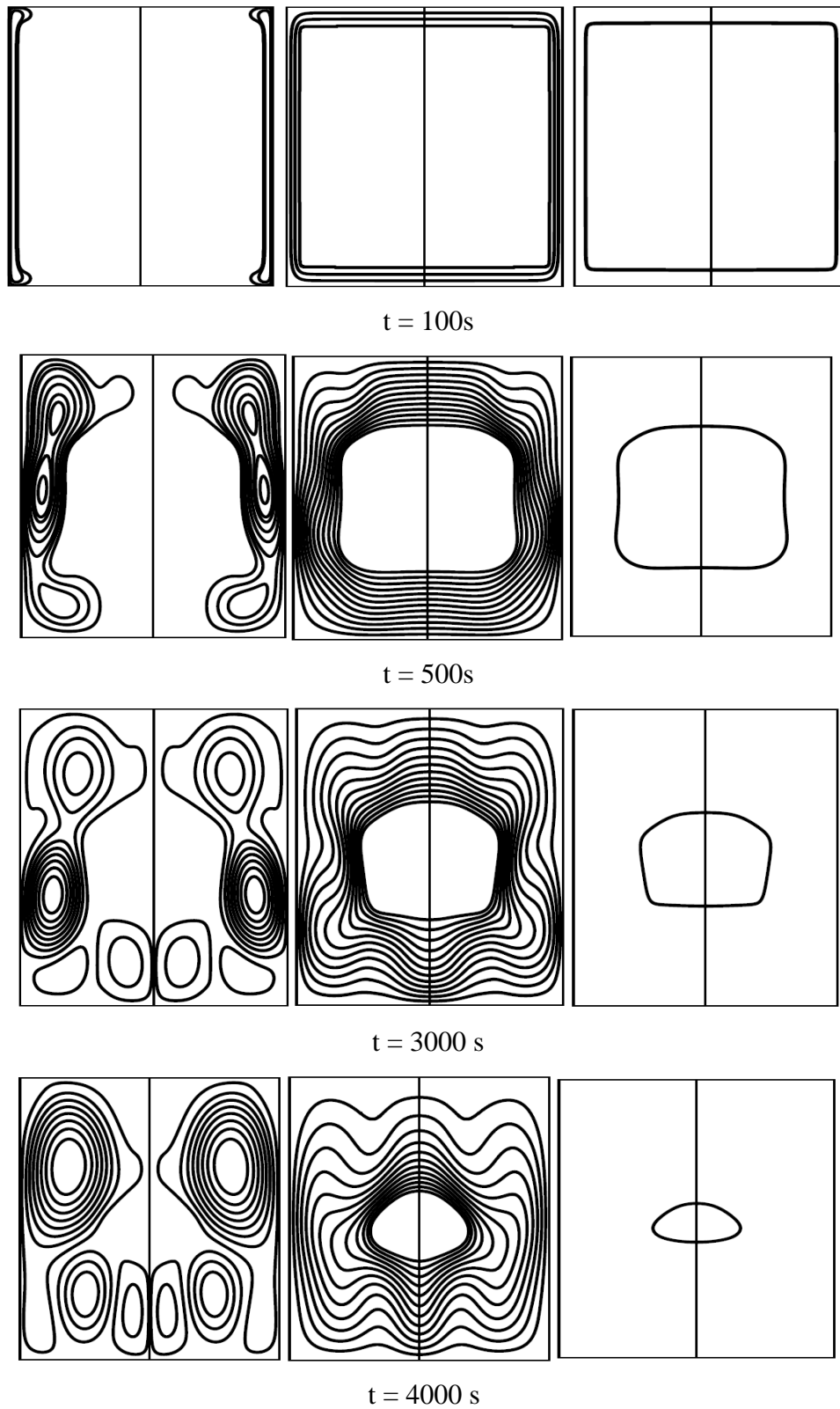
Configuration A

Fig.30: Time evolution of the streamlines, isotherms and the interface solid-liquid for

$$Ra = 47390$$

At the first stages $t \leq 500s$, the conduction is the predominated heat transfer mode, where the isotherms are parallel to the heated walls, and the solid-liquid interface moves uniformly inward from the surface of the capsule. The isotherms at the horizontal surfaces of the cavity remain parallel. The heat is transmitted through the melt from the top heated surface to the bottom cold melt in the upper part of the capsule. Consequently, lighter fluid layers remains frozen on fluid heavier layers and the flow is stagnant in this region. On the other side, heat is transmitted through the melt from the bottom heated surface to the top cold interface in the bottom region of the capsule. In this case, the situation is unstable because of the presence of cold and denser fluid layers adjacent to the solid-liquid interface which lie above hot and lighter fluid layers near the bottom heated wall. A weak convective recirculating flow has already established in the melt layer near the vertical heated wall.

A Bénard clock wise recirculation cell appears at $t = 2000s$, after $t = 3000s$ a second counter rotating Bénard cell appears in the bottom layer. As melting progresses, the Bénard cell at the right, being influenced by the lateral counterclockwise recirculation roll, grows faster and stronger than the left one.

To see how convection affects overall conduction through the cylindrical capsule, the Nussel number is a good indicator, it is defined as:

$$Nu = \frac{h \times L_{ref}}{\lambda} \quad (16)$$

Where L_{ref} is the reference length, h is the heat transfer coefficient and λ is the thermal conductivity, where:

$$h = \frac{q}{T_{wall} - T_{ref}} \quad (17)$$

T_{wall} is the wall temperature, T_{ref} is the reference temperature and q is the convective heat flux. The local Nussel number at the top, the bottom and at the side of the capsule for various instant are presented in Fig.31.

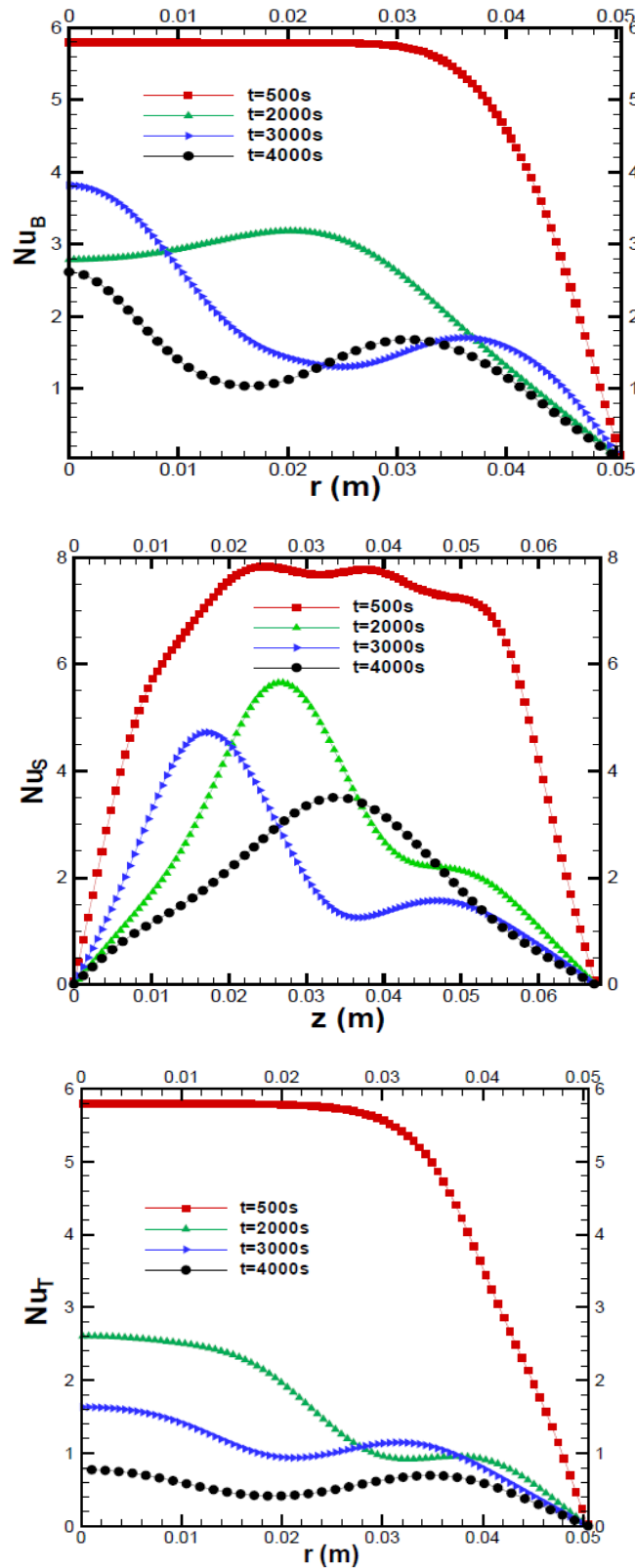


Fig.31: The local Nusselt number at the bottom (Nu_B), at the vertical (Nu_S) and at the top (Nu_T) surface of the cylinder for $Ra = 47390$

The results show the conduction is the predominant heat transfer at the top heated surface; as melting progressed, the Nusselt number on the top surface decreases. the side surface contain he highest heat transfer rates.

Fig.32 displays the streamlines and isotherms in function of time for $Ra = 269534$ the remaining parameters are kept unchanged.

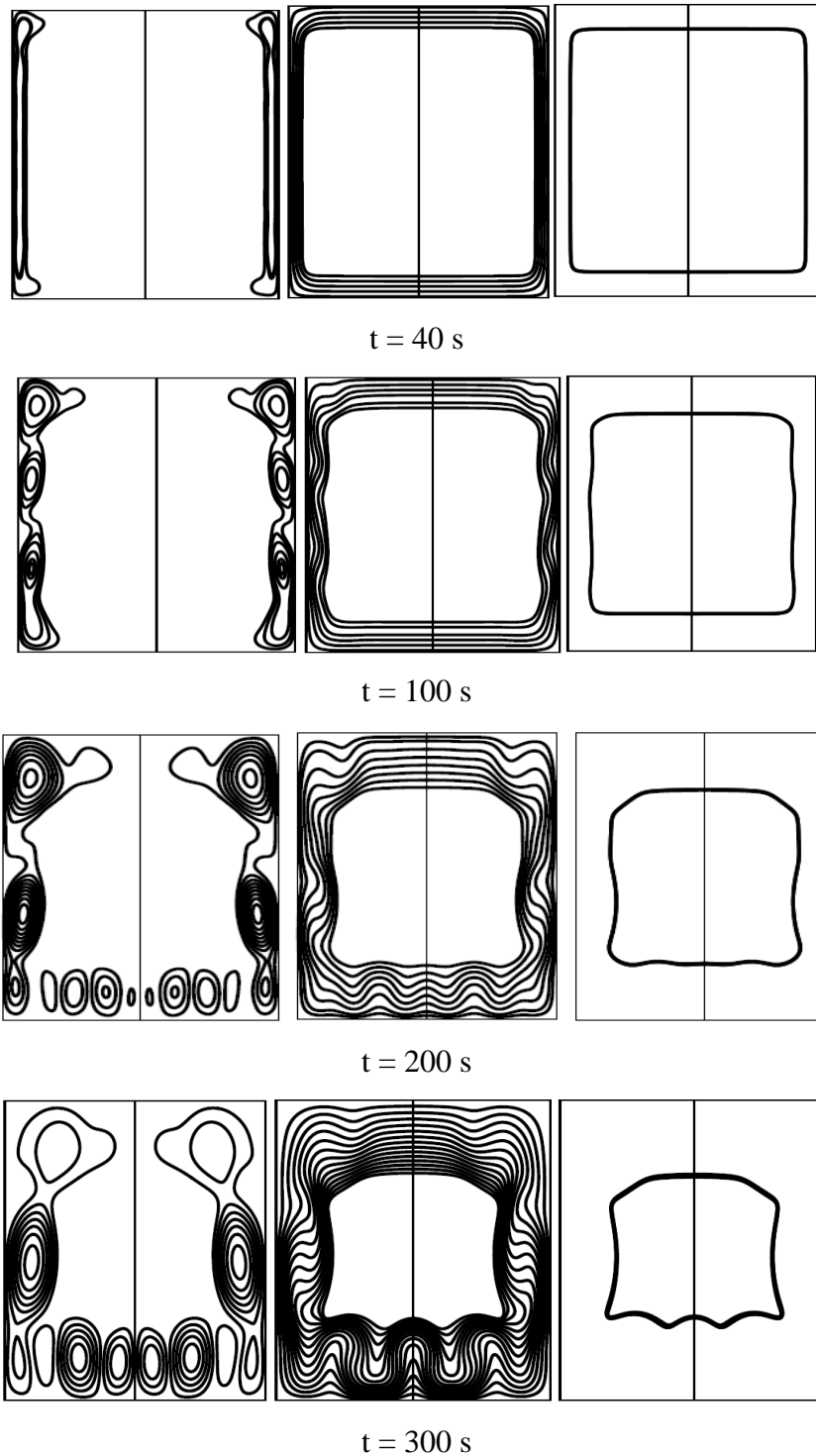


Fig.32: Time evolution of the streamlines, isotherms and the interface solid-liquid for

$$Ra = 269534$$

The results show that Bénard convection appears much earlier (at $t = 200\text{s}$) due to the relatively stronger convective motion inside the melt layer along the vertical heated wall. As melting progresses, an expansion of the melt layer at the bottom appears, and the Bénard cell at the right grows faster than the left one.

The evolution of the liquid fraction inside the capsule is presented in Fig.33.

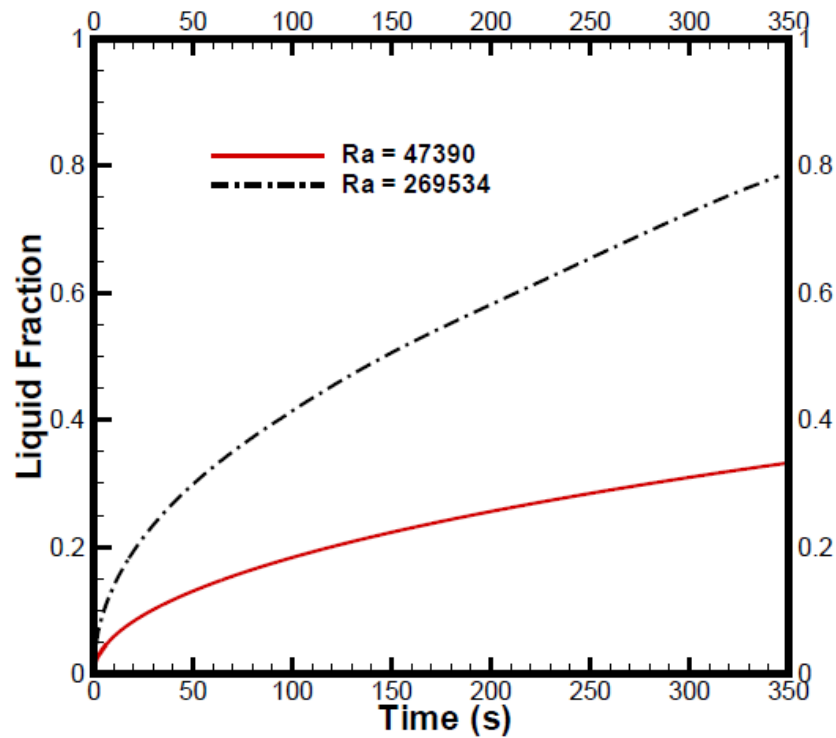


Fig.33: The liquid fraction inside the cylindrical capsule versus time.

It is found that the evolution of the liquid fraction has a linear profile as the convective motion throughout the melt is well established.

- Configuration B

Fig.34 displays the evolution of the isotherms, the streamlines, and the interface solid-liquid inside the capsule for a case with $Ra = 47390$.

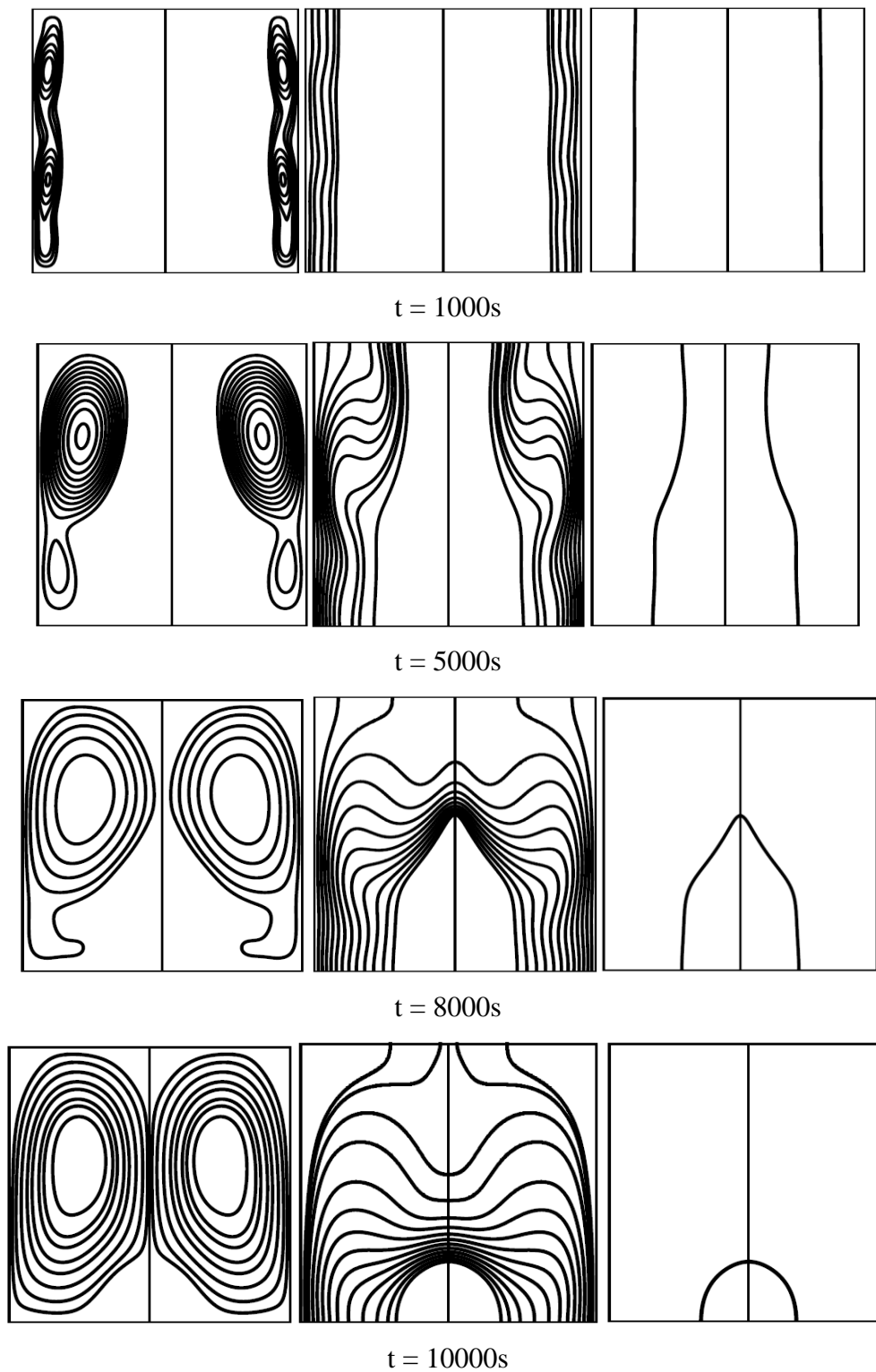


Fig.34: Time evolution of the isotherms , streamlines, and the interface solid-liquid for $Ra = 47390$

The results show three different regimes: pure conduction, mixed convection/conduction and convection dominant. At first stages, a uniform thickness of the molten layer characterize the z-direction, Fig.34 ($t = 1000$ s). Because of the small molten

layer thickness, conduction is the dominant heat transfer mechanism. As the melting progresses, the molten layer being to become thickest at the top of the enclosure as observed in Fig.34 ($t=5000s$), indicating that the buoyancy –driven natural convection currents are beginning to strengthen, transporting hot fluid toward the top of the cylindrical capsule. At this stage, conduction is still an important mechanism, since the melt layer thickness is still nearly uniform over much of the cylinder height. The effects of the natural convection currents on the solid/liquid interface become more pronounced, in this stage the melt layer thickness varies continuously along the height of the cylinder as seen in Fig.34 ($t = 8000 s$).

At $t = 10000 s$, the top portion of the PCM has completely melted away. We can conclude that the process of melting in the vertical cylindrical capsule is influenced by both convection and conduction, The convection determines the characteristic conical shape of the remaining solid [18]. We can also note that, while heating the cavity from the bottom, side, and top surfaces simultaneously, the convective flow patterns and time evolution of the phase front are more complicated than those for the melting from a single isothermal boundary.

Fig.35 presents the time evolution of the isotherms and streamlines for $Ra = 1380254$, the other parameters are kept unchanged.

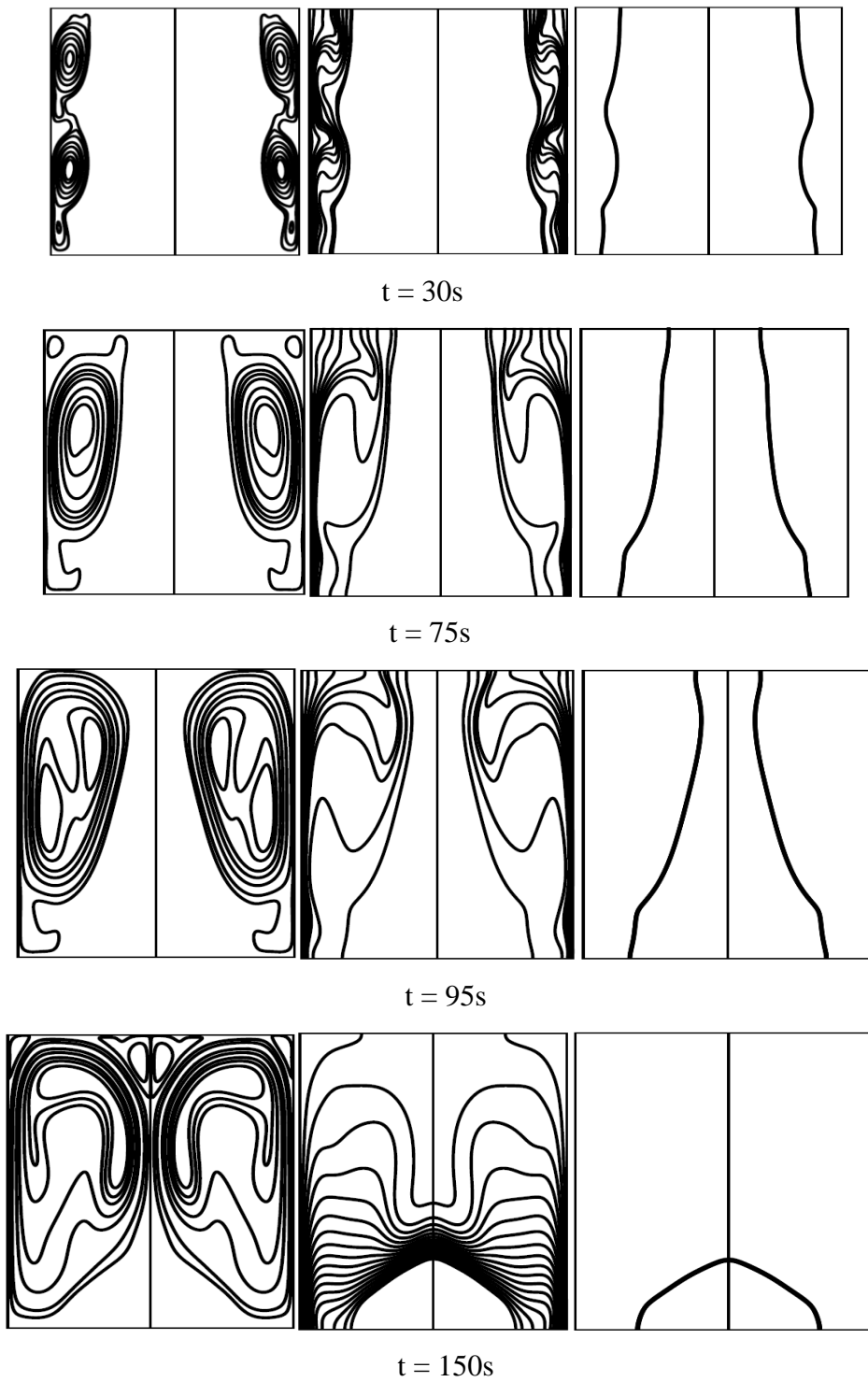


Fig.35: Time evolution of the streamlines, isotherms and the interface solid-liquid for

$$Ra = 1380254$$

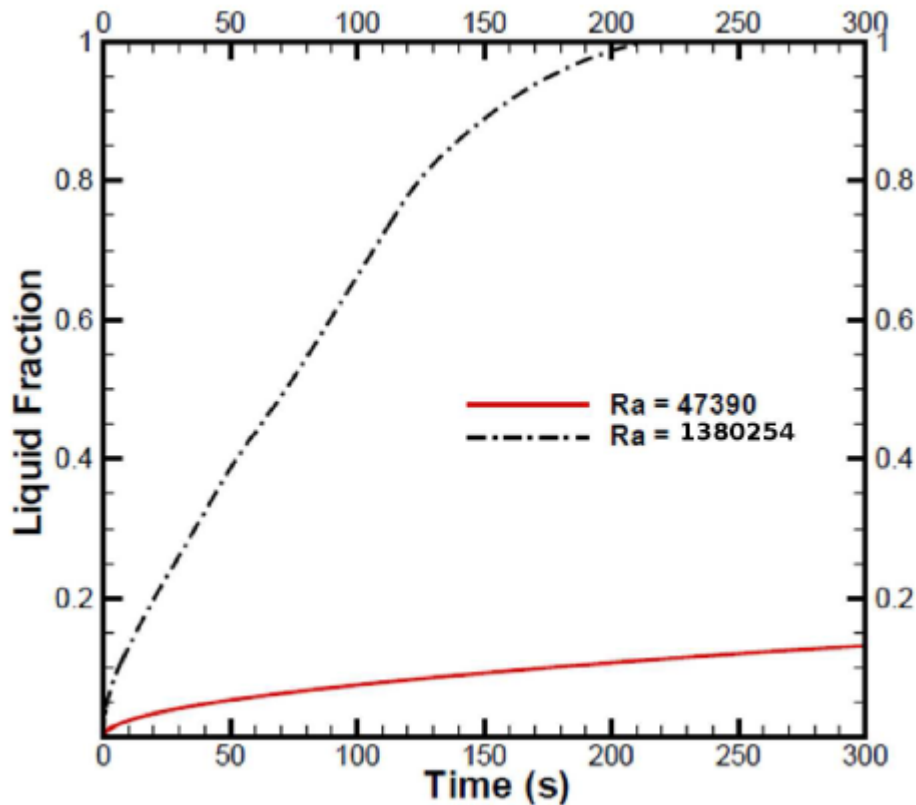


Fig.36: Temporal variation of the liquid fraction inside the cylindrical capsule

We can note here that natural convection occurred much earlier time (at $t = 30\text{s}$) due to the relatively stronger convective motion inside the melt layer along the vertical heated wall.

The temporal variation of the liquid fraction inside the capsule is presented in Fig.36 for $Ra = 1380254$. For this Rayleigh number the liquid fraction does not linearly increase with the time especially at the last part of the melting process

a. Conclusion

A physical model using an enthalpy porosity technique on an unstructured finite-volume method is developed to investigate the natural convection effect on the melting process within a vertical cylinder. Numerical results corresponding to the melting process under two work conditions are presented. Results show that the convective flow patterns and time evolution of the phase front resulting from simultaneous bottom, side and top heating, are more complicated than those for the melting from a single isothermal boundary. The resulting melt shapes and the temperature in the PCM provide conclusive evidence of the importance of natural convection on heat transfer in the melt region.

2- Melting of PCM in a horizontal cylinder

a. Introduction

Due to its relevance in many technological applications like in latent-heat energy storage systems, solar energy systems, casting and crystal growth processes [1,2], heat transfer during freezing and/or melting of a phase change material (PCM) has attracted considerable attention over the past decades. Thermal energy storage is very important in storing the excess of energy which would be otherwise wasted, and in matching supply and demand. Because of the relatively high energy storage density and the possibility of storing and delivering latent heat at nearly constant temperature, latent heat thermal energy storage (LHTES) systems are becoming increasingly. Unfortunately, the low thermal conductivity of many PCMs, consist one of the main disadvantages which is leading to poor melting and solidification rates. Therefore, good understanding of heat transfer in the PCM during the phase transition is needed to the advance in LHTES.

A lot of studies have examined various aspects of melting and freezing phenomena both from the fundamental as well as applications point of view [3,4]. Different geometrical encapsulations have been proposed for the PCM enclosures: tall rectangular enclosures, spherical shells, and cylindrical pipes [1]... .

The non-linearity of the governing energy equation and a wide variety of geometric and thermal boundary conditions provide a fertile ground for challenging basic research problems [5,6]. Also, numerous industrial applications in diverse industries provide the necessary incentive for engineering research and development [7,8]. Time-dependent boundary conditions, under some conditions can lead to interesting and unique multiple moving boundaries as well. Although a number of numerical and experimental investigations have been devoted to convection dominated melting of a PCM for various encapsulations, special attention is devoted to melting in a horizontal cylinder for thermal energy storage system application [9–16]. A number of numerical and analytical studies have been devoted to model the melting phenomenon in a horizontal cylinder based on the Boussinesq approximation [17–23]. Saitoh and Hirose [30] presented experimental results showing convexed solid-liquid interface at the bottom of the unmelted solid during the melting process.

Rieger et al. [31], Ho and Viskanta [32] and Yoo and Ro [33] studied experimentally the melting process of a PCM contained in a horizontal cylinder in term of the evolution of

the solid-liquid interface. Their results showed concaved interface numerically and experimentally. Prasad and Sengupta [34] realize a numerical investigation on the unconstrained melting of a PCM inside an isothermal horizontal tube. Their model show irregular temporal shape of the solid-liquid interface due to density difference, and natural convection in the liquid phase. Park et al. [35] realize a numerical investigation of phase change material melting inside a horizontal cylinder heated isothermally was presented. An initial disturbance was introduced, and its effect on the solution is studied for two Rayleigh number values. The results show that at $Ra=106$, the flow was unicellular, but $Re = 8.106$, two branching solutions (bi-cellular and tri-cellular flow) were observed depending on the type of initial disturbances. Ro and Kim's [36] showed, by using the numerical analysis, that both interface shapes are possible. In [37] Chung et al. analyzed numerically the multi-cell structure and the thermal instability at the early stage of the melting process in a horizontal cylinder heated isothermally for a relatively wide range of the Rayleigh numbers by the use of the enthalpy method. The results stated that the flow in the liquid gap is in the stable state as the viscous force dominates for the low Rayleigh number. For high Rayleigh number, the Bénard convection is not affected by the base flow and shows an orderly behavior.

This section aims to investigate numerically the natural convection dominated melting of a PCM inside a horizontal cylindrical cavity for Rayleigh numbers in the range $10^4 < Ra < 10^8$. The enthalpy-porosity method is used where the interface solid-liquid is implicitly determined in the calculation domain.

Fig. 37 describes the problem considered in this section: melting process of a PCM at initial temperature of T_i in a horizontal cylinder whose wall temperature is T_w above the melting temperature T_m . The gallium is chosen as the test PCM.

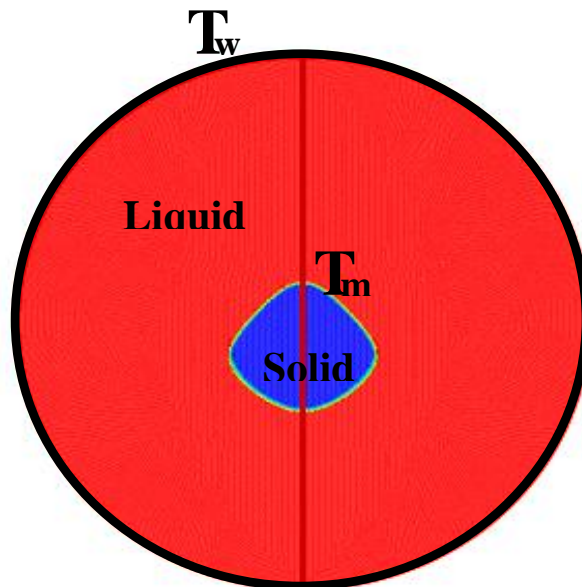


Fig. 37: melting process inside a horizontal cylinder

b. Results and discussion

Simulations were carried out for melting of a PCM (Gallium) within a horizontal cylinder. Numerical investigations were conducted using 85,000 cells and the time step of 10⁻³ s was found to be sufficient to give accurate results. The effect of the melting process on the streamlines, temperature contours and the solid-liquid interface for different Fourier number is gauged through the result illustrated in Fig. 38.

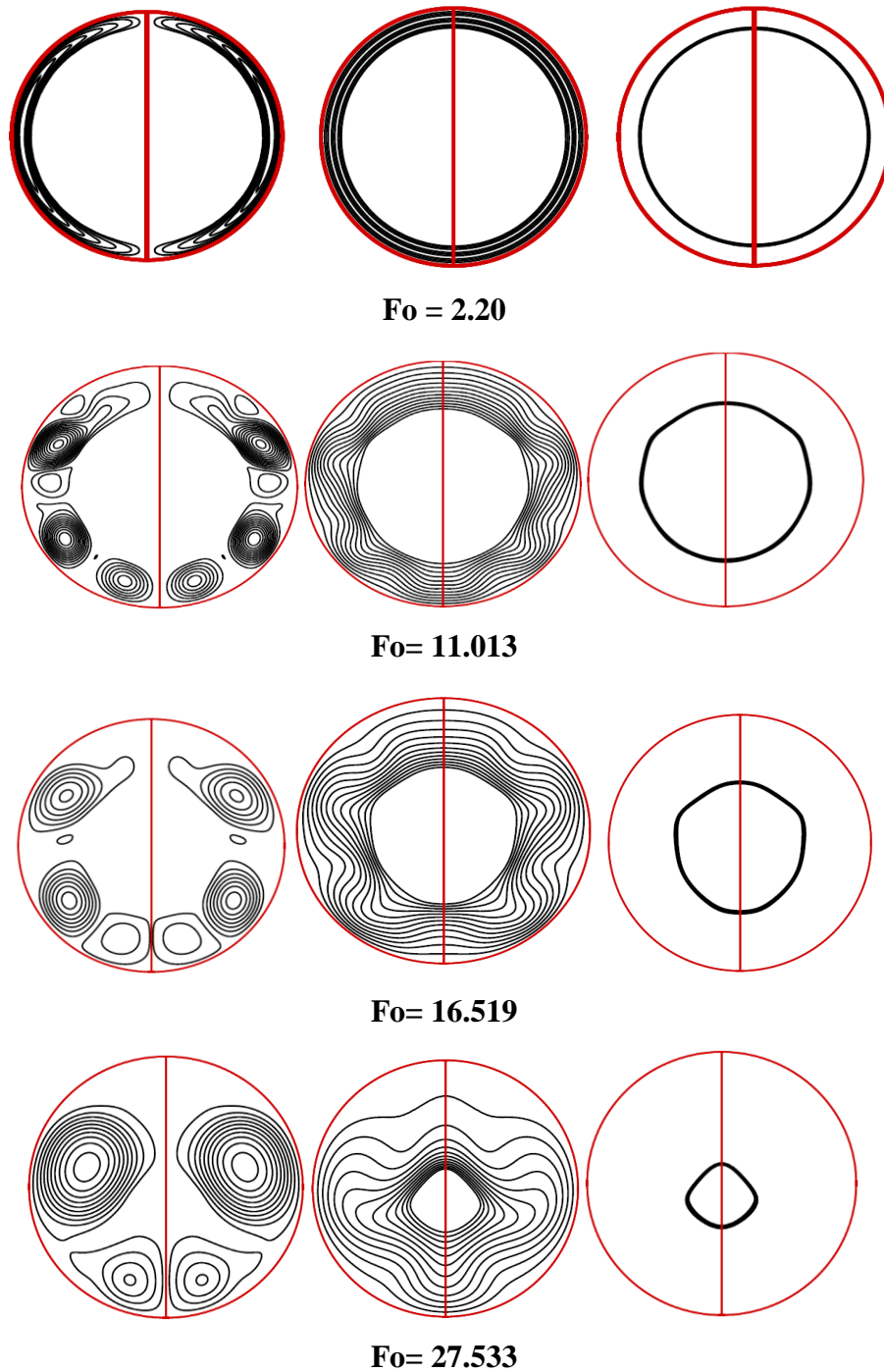


Fig. 38: Stream lines, temperatures contours and interfaces liquid-solid at various times
($Ra = 3.10^4$)

During the early phase the liquid is confined between the rigid heated cylinder and a concentric moving solid-liquid interface. Inspection of the figures reveals that at early times the melt regions are similar in shape and that when heat transfer to the gallium is predominantly by conduction the melt region is symmetrical about the axis of the cylinder. After some time natural convection develops and intensifies, influencing the melt shape in

general and the melt region above the cylinder in particular. Natural convection conveys the hot liquid to the upper part of the melt zone and in this manner continues to support the upward movement of the solid-liquid interface. At low Rayleigh number ($Ra = 3.10^4$) the solid-liquid interface is approximately concentric inside the cylinder (see Fig. 38). We can also note that during the melting process the flow pattern changes totally as in Fig.38 which shows the flow transition from the base flow (single-cell) to the three-cell flow at the intermediate stage and finally to the two cell flow as time elapses. Figs. 39-a and 39-b display the flow behavior during the melting process for Rayleigh numbers of 5.10^6 and 4.10^7 respectively. After the conduction dominating stage, a multi-cellular flow patterns characterize by a complex structure of the fluid dynamics. The number and the size of the cells depend on the size of the molten phase.

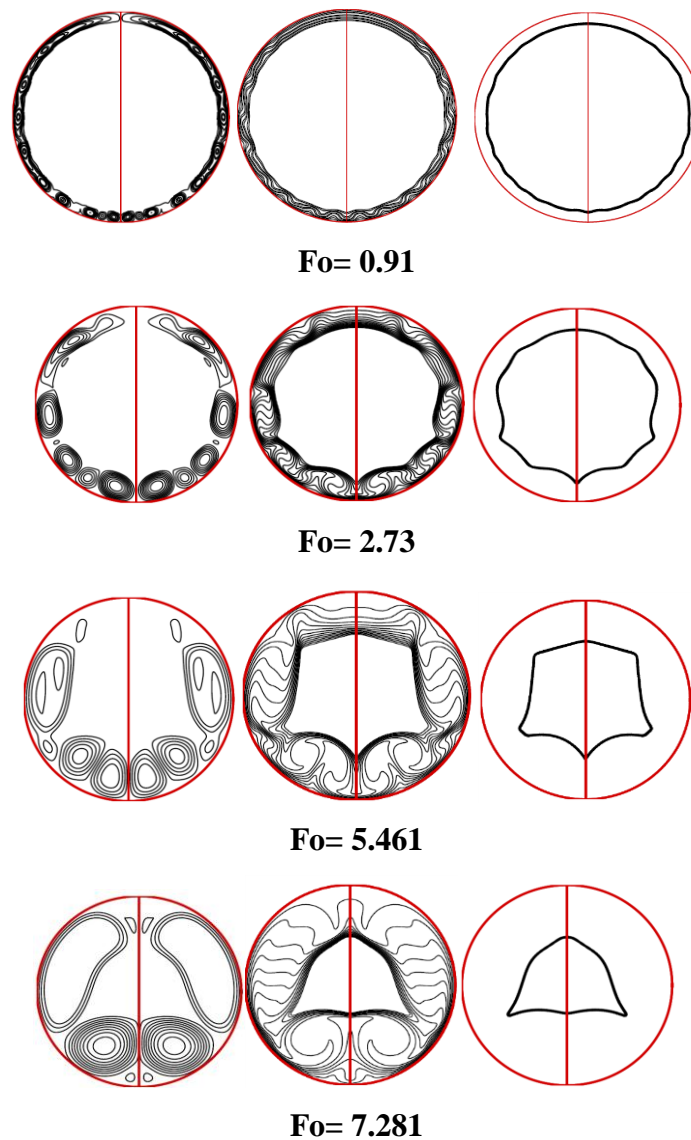


Fig. 39-a: Stream lines, isotherms and interfaces liquid-solid at various times
($Ra = 5.10^6$)

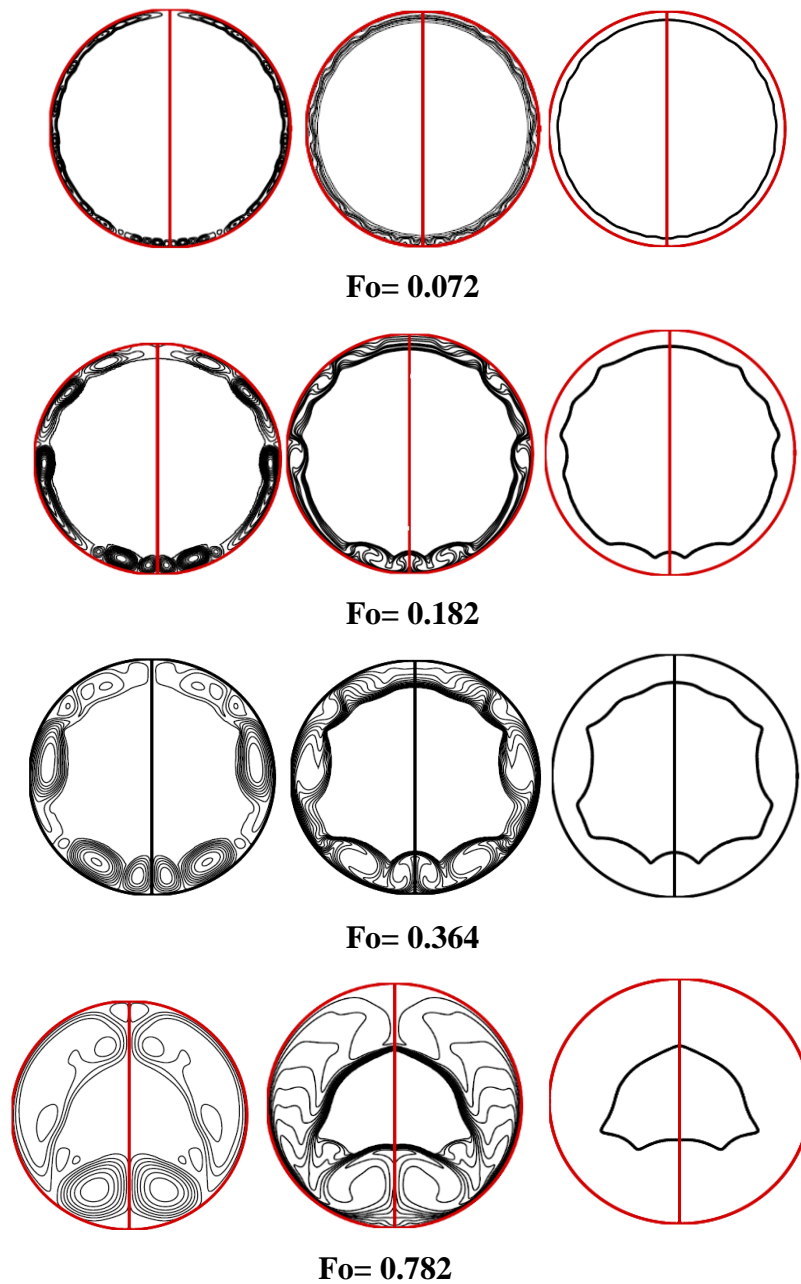


Fig. 39-b: Stream lines, isotherms and interfaces liquid-solid at various times
($Ra = 4.10^7$)

For $Fo=5.461$ and $Ra = 5.10^6$, we observe the presence of four counter-rotating cells (see Fig. 38). As the liquid layer increases, the number of rolls decreases. So at $Fo=7.281$, when nearly 50% of the PCM have been melt, two counter-rotating cells are obtained. This is very similar to the Bénard convection. As the liquid gap between the solid-liquid interface and the cylinder bottom wall is narrow enough, it can be approximated as two flat plates. This

phenomenon leads then in the sequel of calculations to the development of the above mentioned roll cells. The establishment of these vortices influences the melting kinetics of the whole system. Especially the heat transfer in the lower part of the melting gap is greatly improved which results in a “moonshaped” melting contour. We can also observe that the flow in the liquid zone remains in the stable state at low Rayleigh number. At the high Rayleigh number, the Bénard convection presents an orderly behavior without been affected by the base flow.

The effect of the Rayleigh number on the local Nusselt number at diferent angular positions along the cylinder surface for several Fourier numbers is shown in Figs. 40, 41 and 42.

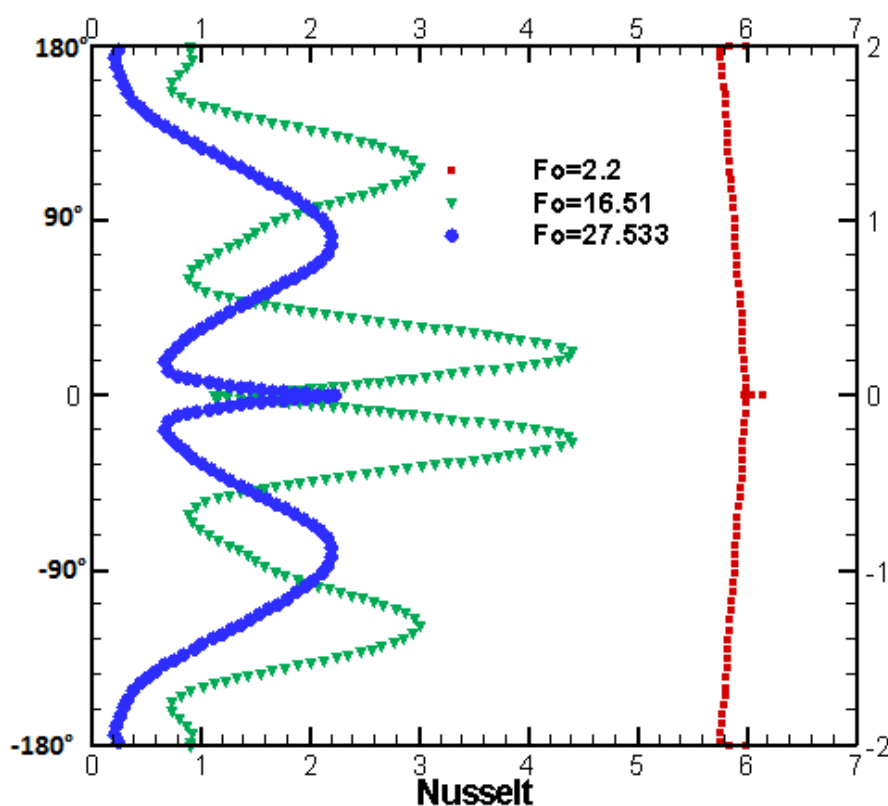


Fig. 40: Local Nusselt number at various times ($Ra = 3.10^4$)

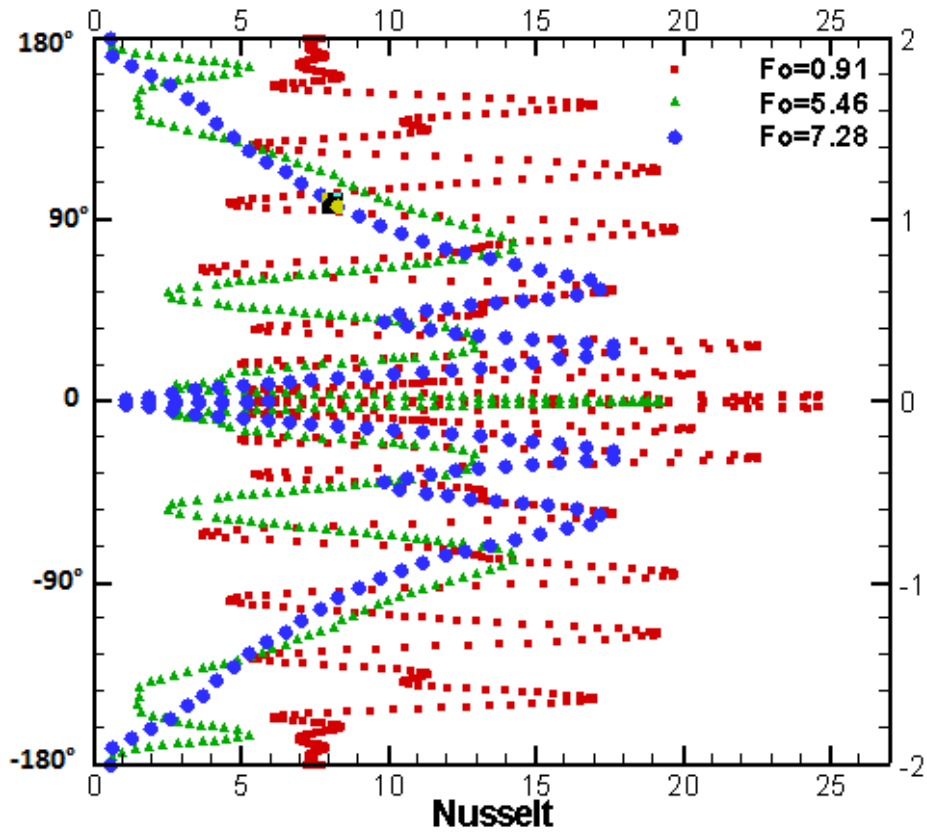


Fig. 41: Local Nusselt number at various times ($Ra = 5.10^6$)

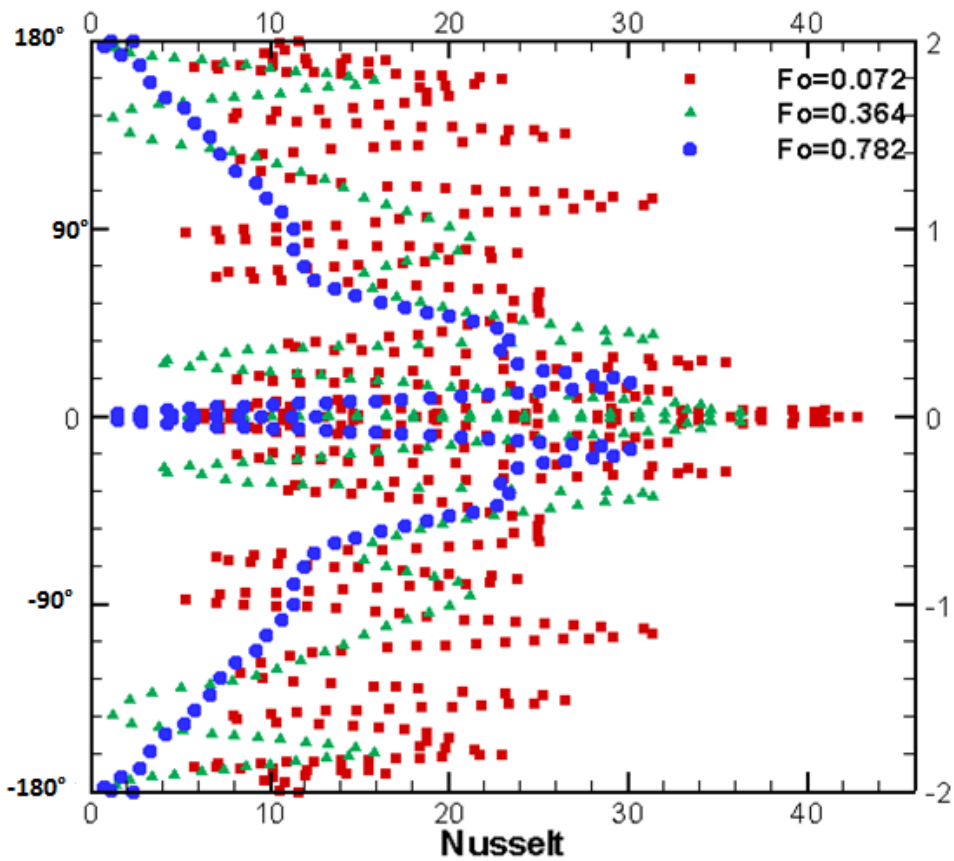


Fig. 42: Local Nusselt number at various times ($Ra = 4.10^7$)

For low Rayleigh number ($Ra = 3.10^4$), the Nusselt number is approximately constant at the first stage of the melting process. This result confirms that at early times, heat transfer in the melt zone predominantly by conduction. Sudden drops of the Nusselt number appear at times corresponding to roll pairings. they are more pronounced for $Ra = 5.10^6$ and $Ra = 4.10^7$. These results can be explained by the fact that the higher Rayleigh number is related to strong natural convection velocities which results in higher heat transfer rates along the cylinder surface. It is interesting to note that the rate of melting is one of the most important parameters of the problem. The time evolution of the total liquid fraction (ratio of volume of melt to volume cavity) is a factor that has been widely used as a monitoring parameter in earlier publications. From the liquid fraction versus time plot, one can get both the rate melting (slope of the tangent line at a given time) and the average melting rate (ratio of current liquid fraction and time).

Fig. 43 shows the time evolution of the total fraction of the liquid in the cylindrical capsule for two Rayleigh numbers. We note that as Rayleigh number increases the PCM inside the cylinder melts faster. We can also note that, once the convection is established, the variation of the liquid fraction with time seems to be linear.

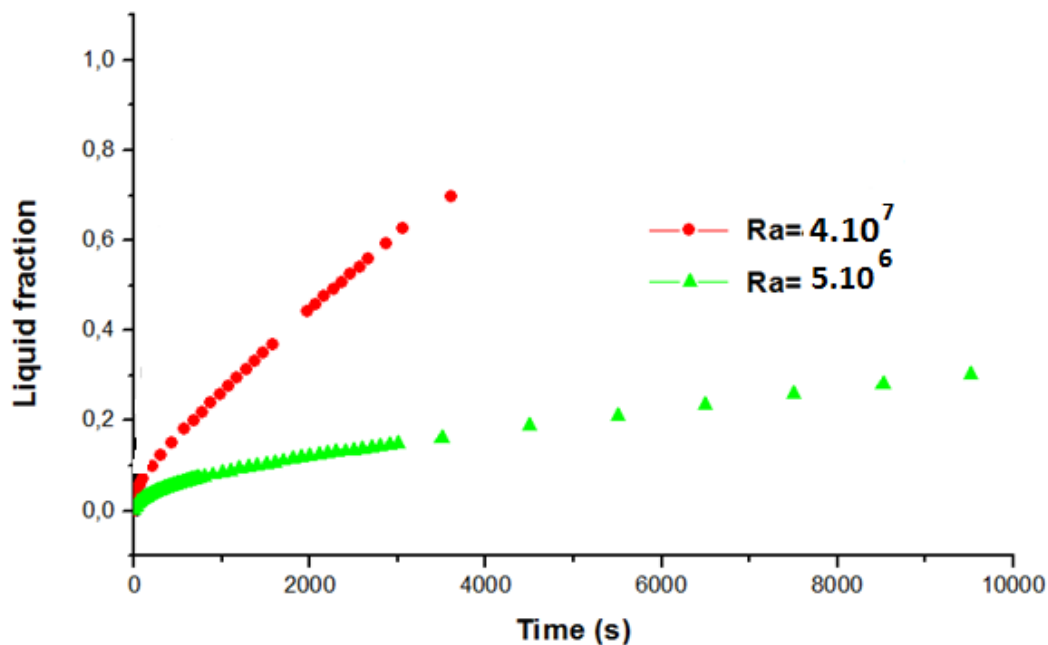


Fig. 43: Liquid fraction for various Rayleigh numbers

c. Conclusion

This section investigates numerically the Rayleigh number effect on the natural convection during the melting process within a horizontal cylinder. It is found that the melting process of PCM is dominated by conduction at early stages indicated by concentric solid–liquid interfaces parallel to the circular heated wall. At later times, natural convection is augmented and this affects the shape of the liquid solid interface and consequently results in unsymmetrical melting about the horizontal axis of the capsule. The numerical results indicate also that for low Rayleigh number, the flow in the melting zone remains in the stable state since the viscous force is dominant. For high Rayleigh number, the Rayleigh-Bénard convection indicates an orderly behavior without being affected by the base flow.

3- Heat transfer in horizontal cylinder with fins embedded in PCM

a. Introduction

Good understanding of heat transfer during melting process is essential for predicting the storage system performance with accuracy and avoiding costly system overdesign [40-41]. Natural convection heat transfer around horizontal circular cylinders finds various practical applications in space heating, heat exchangers, solar energy collectors, energy storage systems, and electronic devices. During the solidification process, conduction is the sole transport mechanism but in the case of melting natural convection occurs in the melt region and this generally enhances the heat transfer rate compared to the solidification process. It is possible to increase the heat transfer during phase change process with different methods, including fins, metal matrices, high conductivity particles or graphite [42-43]. The use of fins to enhance heat conduction in latent energy storage systems is investigated experimentally and numerically by numerous researchers [44-48].

The aim of this section is to point out some of the natural convection characteristics during the melting process of PCM around a circular cylinder with three rectangular fins. The contribution of the present study to the actual knowledge lies in the fact that the problem of the melting process is formulated using the enthalpy-porosity based method. A numerical code is developed using an unstructured finite volume method. Melting temperature, the

thermal conductivity and the latent melting of PCM on the thermal behavior of PCM building materials is also presented.

b. Results and discussion

Using the model describe in chapter 1, simulations were carried out for melting of a PCM around a horizontal cylinder with and without rectangular fins. The computational domains of the physical model are shown in Fig.44. The inner cylinder tube (inner radius $R_i = 2.7$ cm) is hot and the outer cylinder tube (outer radius $R_e = 7.6$ cm) is insulated. The solid PCM was subcooled at $TS = 302$ K while the wall temperature T_w of the inner cylinder fixed to the 310 K. The brass fins have a cross-section of $0.6 \text{ cm} \times 2.7 \text{ cm}$. Numerical investigations were conducted using 45000 cells and the time step of 10^{-4} s was found to be sufficient to give accurate results.

The fins increase the heat source surface and the volume by 190% and 76%, respectively. In order to compare the numerical results for a cylinder without and with extended surfaces, the diameter of the cylindrical part of the finned cylinder was chosen to be the same as the diameter of the bare cylinder.

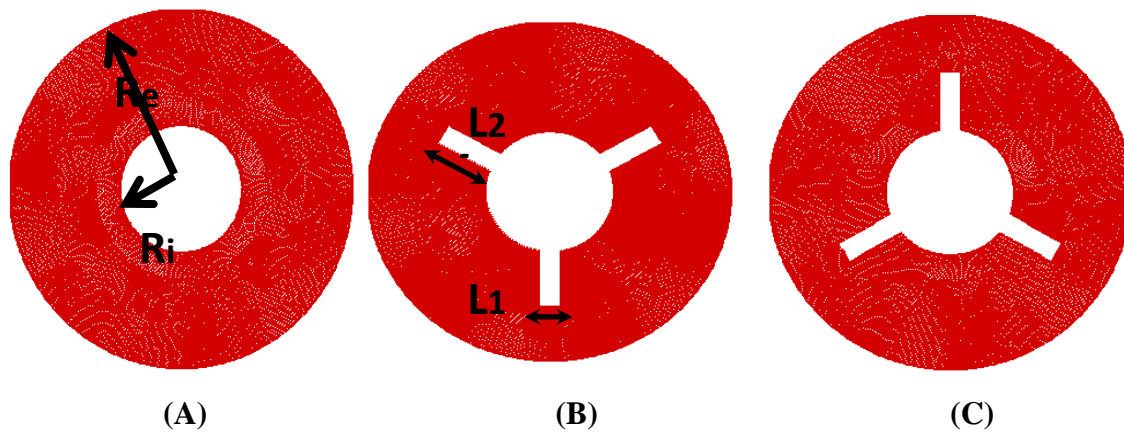


Fig.44: Layouts of the physical model and correspondent meshes

The effect of the melting process on the streamlines, temperature contours and the solid-liquid interface for several times is gauged through the result illustrated in Fig.45.

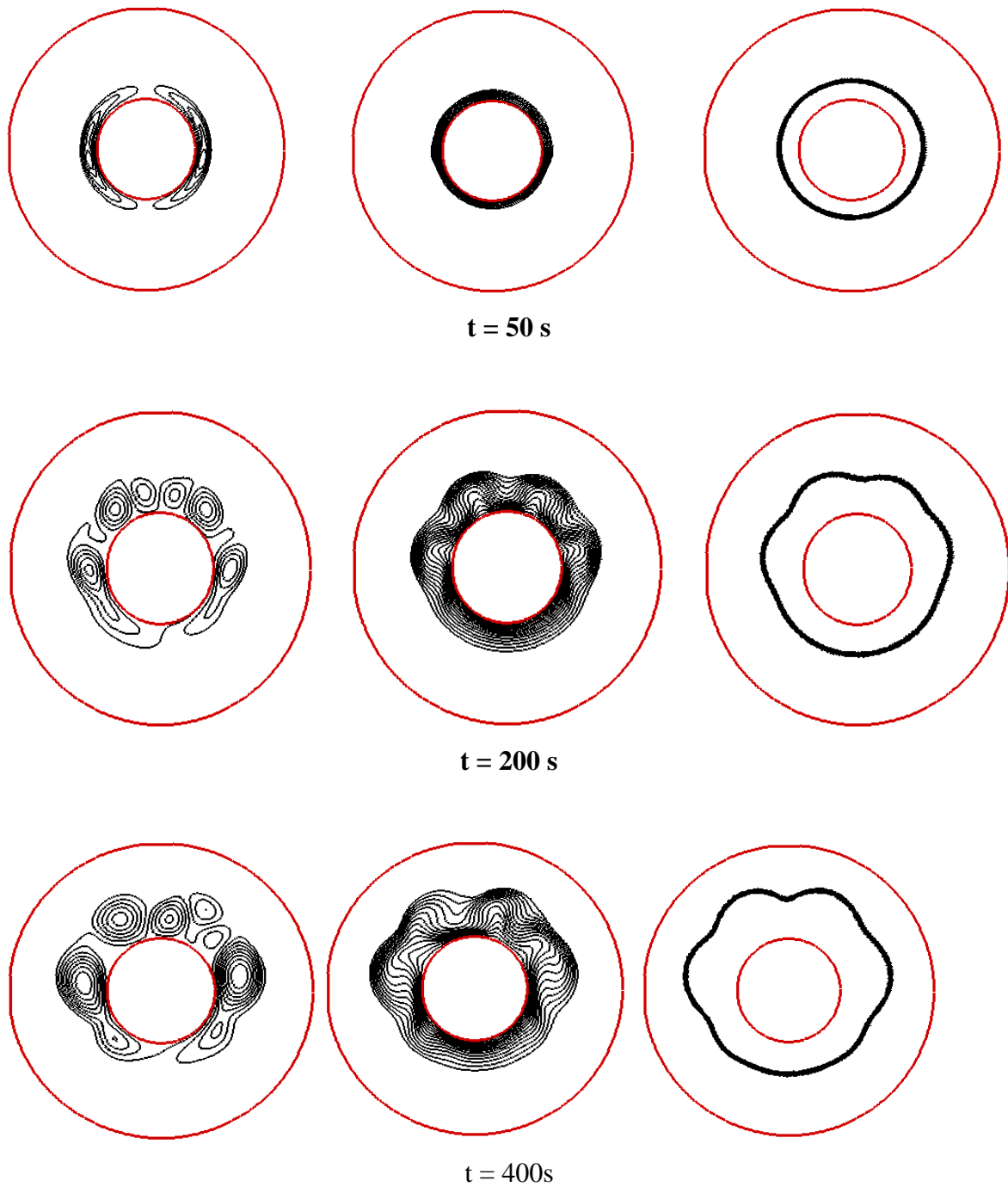


Fig.45: Stream lines, temperatures contours and interfaces liquid-solid at times 50s, 200s and 400s (constant wall temperature $T_w=310$ K)

During the early phase the liquid is confined between the rigid heated cylinder and a concentric moving solid-liquid interface. Inspection of the figures reveals that at early times the melt regions are similar in shape and that when heat transfer to the gallium is

predominantly by conduction the melt region is symmetrical about the axis of the cylinder. After some time natural convection develops and intensifies, influencing the melt shape in general and the melt region above the cylinder in particular. Natural convection conveys the hot liquid to the upper part of the melt zone and in this manner continues to support the upward movement of the solid-liquid interface.

Due to the extension of the fins in vertical direction, the characteristic length for the finned cylinder should be different than that for the cylinder without fins. It appears from Figs. 46 and 47 that for very short times the heat transfer is essentially by conduction, as evidenced by a liquid layer of constant thickness around the heat source. Natural convection was observed first to occur at the top of the finned heat exchanger and was due to the thermal instability induced by the temperature differences along the fins and the hot surface of the cylinder. As in the case of cylinder without fins, there was more melting above the heat source and very little below. This is clearly demonstrated by the results of Figs 46 and 47. The strong upward thrust of the melting zone above the finned cylinder is caused by natural convection.

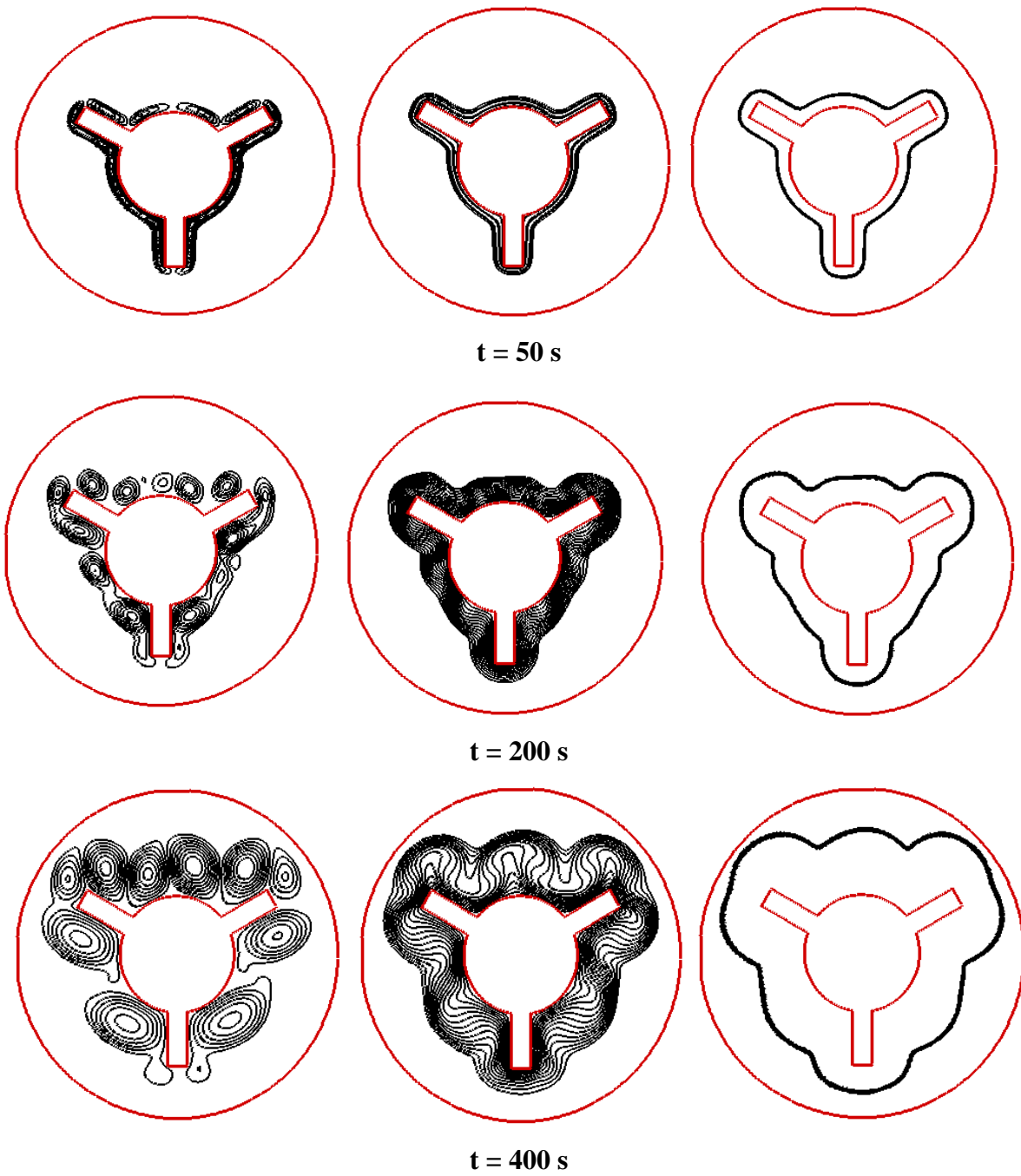


Fig.46: Stream lines, temperatures contours and interfaces liquid-solid at times 50s, 200s and 400s (constant wall temperature $T_w=310$ K)

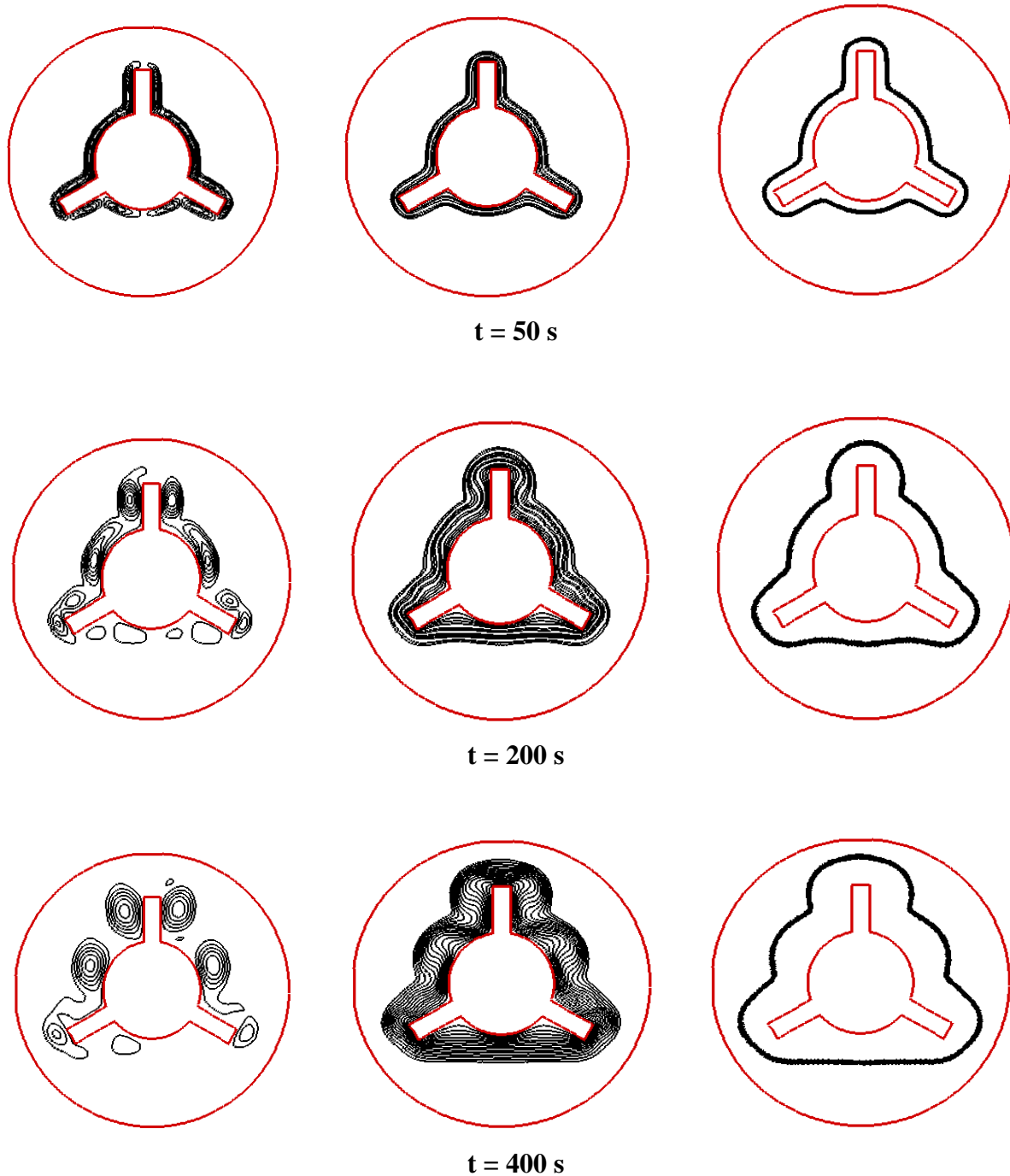


Fig.47: Stream lines, temperatures contours and interfaces liquid-solid at times 50s, 200s and 400s (constant wall temperature $T_w=310 \text{ K}$)

Fig. 48 displays the time evolution of the total fraction of the liquid in the cavity versus Fourier number F for the three configurations (A, B and C). The volume of the PCM melted in the two arrangements of the finned cylinder was higher than for the bare cylinder. As can be seen, arrangement B proved to be more effective than arrangement C. This is due to

the more intense natural convection circulation in the melt. For arrangement C the stable temperature gradients along the fins and the hot cylindrical surface produced a gravitationally stable liquid region between the two lower fins where heat transfer was by conduction only.

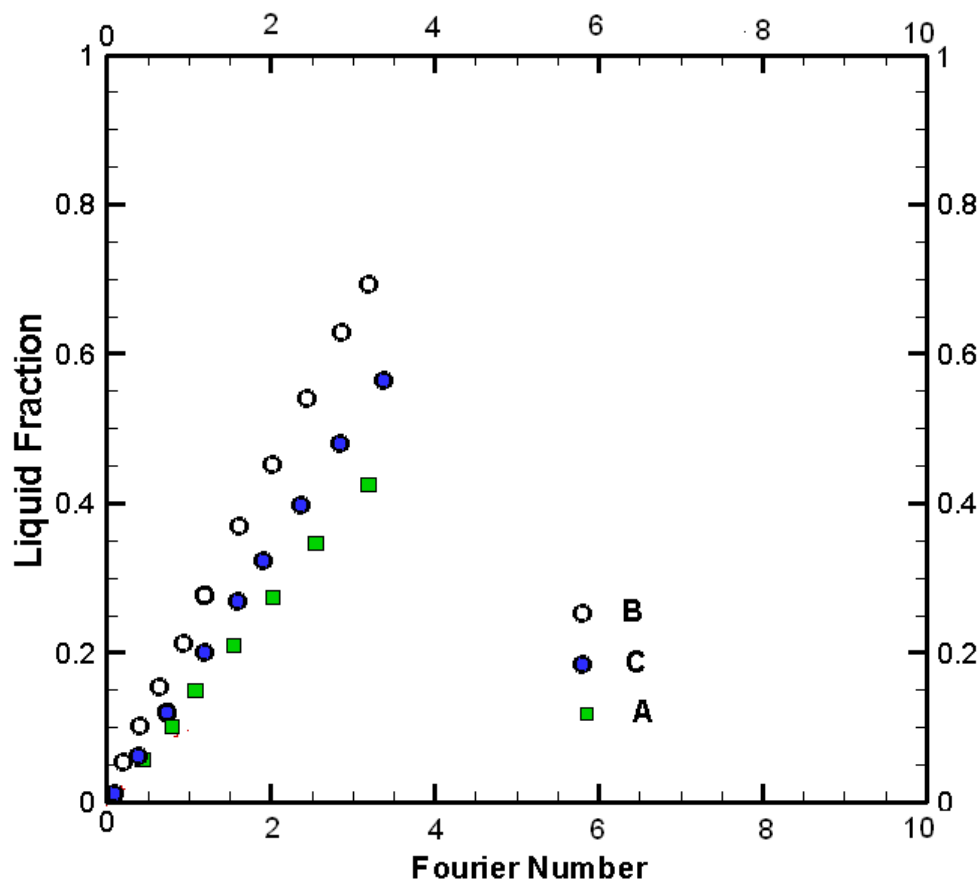


Fig.48: Liquid fraction around cylinder with and without fins

c. Conclusion

Melting process around a horizontal cylindrical heat exchanger with longitudinal fins embedded in a gallium has been studied numerically. Based on the numerical results obtained it has been concluded that:

- The use of extended surfaces increases the rate of melting in comparison to that without fins.
- Orientation of fins with respect to the gravitational field on a horizontal cylindrical is interesting for taking full advantage of natural convection effects, to enhance melting and not to create quiescent melt regions where natural convection circulation may be suppressed.

4- References

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Chapter IV:

Melting of PCM Around Horizontal Cylinders

1- Introduction

In this section, we investigate numerically the heat transfer during the melting process of PCM around a multiple horizontal heat sources. This study is motivated by the need to gain improved understanding of heat transfer during the charging phase of TES system which takes advantage of latent heat-of-fusion of PCM. A relevant consideration in such systems is the effective utilization of the PCM by an optimum arrangement of tubes through which the working fluid is circulated. Good heat transfer characteristics between the transport fluid and the PCM for efficient thermal performance of a storage unit are also required [21, 28]. Natural convection is also an important process in problems involving melting [29–32], and it is the purpose of this section to point out some of its characteristics.

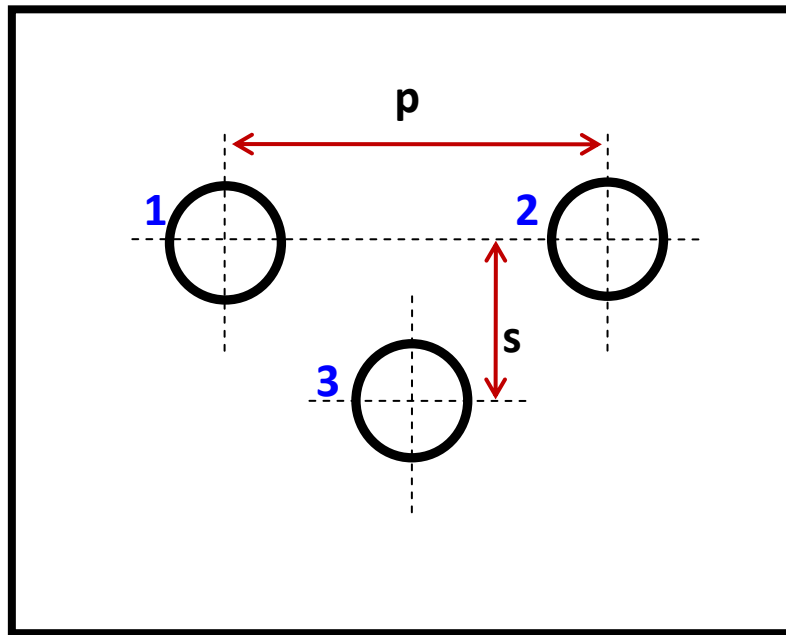
2- Results and discussion

Simulations were carried out for melting of a PCM (Gallium) around three horizontal cylinder arrangements. Two different arrangements (see Fig. 49) have been considered to examine the effects of natural convection during the melting process on the heat transfer and the melting front shape. The pitch and the spacing used are given in Table 2.

The solid PCM was subcooled at $T_s = 302 \text{ K}$ where the wall temperature of the inner cylinder is fixed at $T_w > T_m$. Numerical investigations were conducted using 75000 cells and the time step of 10^{-4} s was found to be sufficient to give accurate results.

Arrangement	Pitch, p	Spacing, s
A ₁	3D	D
A ₂	3D	1.5D
B ₁	3D	D
B ₂	3D	1.5D

Table 2: Cylindrical heat source arrangements used in tests: D= 0.02m



(A)

Fig.49: Cylindrical heat source arrangements used in tests

The influence of the melting process on the streamlines, temperature contours and the solid-liquid interface for various times for arrangement (A1) is illustrated through the result illustrated in Fig. 49. Most of the melting occurs above and to the sides of heat sources with very little below. At early times the liquid is confined between the rigid heated cylinder and a concentric moving solid-liquid interface. Inspection of the plots reveals also that at early times the melt regions are similar in shape and that when heat transfer to the gallium is predominantly by conduction the melt region is symmetrical about the axis of the cylinder. After some time natural convection develops and intensifies, influencing the melt shape in general and the melt region above the cylinders in particular. Natural convection in the melt supports the phase change process as melting continues particularly above the heat source. However, as long as the individual melt regions have not merged to form a common liquid region around the cylinders with a common solid-liquid interface, each melt region develops independently and is not influenced by the presence of other heat source.

Natural convection generates an important upward push of the melting zone. At the first stages of natural convection a plume ascends from the top of the heated cylinder and thereafter circulation sends the hot liquid to the highest part of the melt zone and goes on, in this way, supporting the upward movement of the solid-liquid interface. While the heating continues and natural convection increases, the annular melt zone becomes more and

more deformed. The increase in the wall temperature can provoke oscillation of the plume above the cylinder (see Fig. 50).

As the wall temperature of the cylinders amplifies, the solid over the cylinder melts quicker. After the liquid regions around the cylinders form a common boundary, the natural convection about the lower cylinder 3 supports melting in the region between and above cylinders 1 and 2. With increasing the wall temperature, the natural convection circulation in the liquid becomes more intense and influences the shape of the melt particularly in the region between and above the upper two cylinders (see Fig. 50).

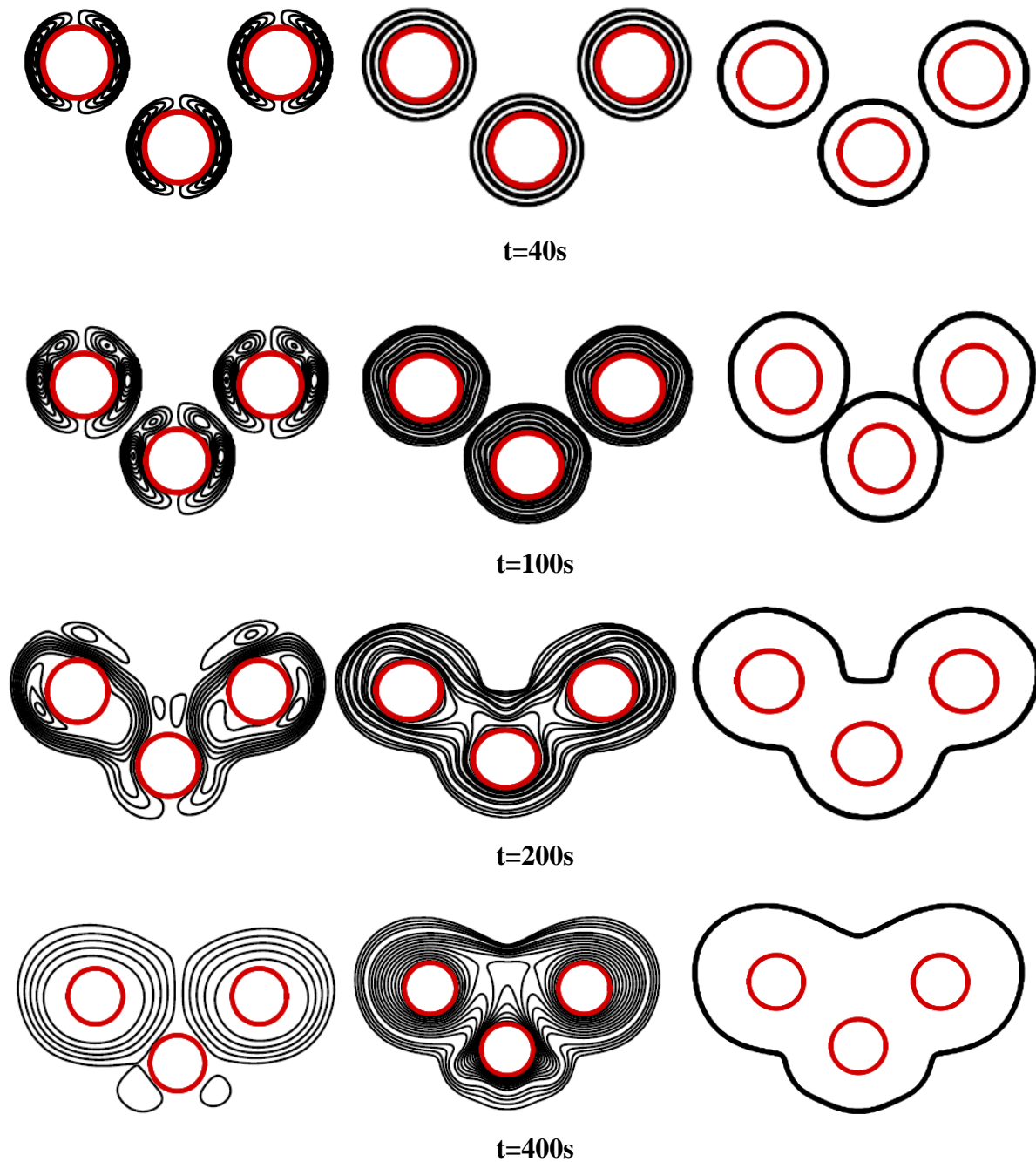


Fig. 50: Stream lines, temperatures contours and interfaces liquid-solid at various times for $T_w=311\text{K}$ (Arrangement A_1)

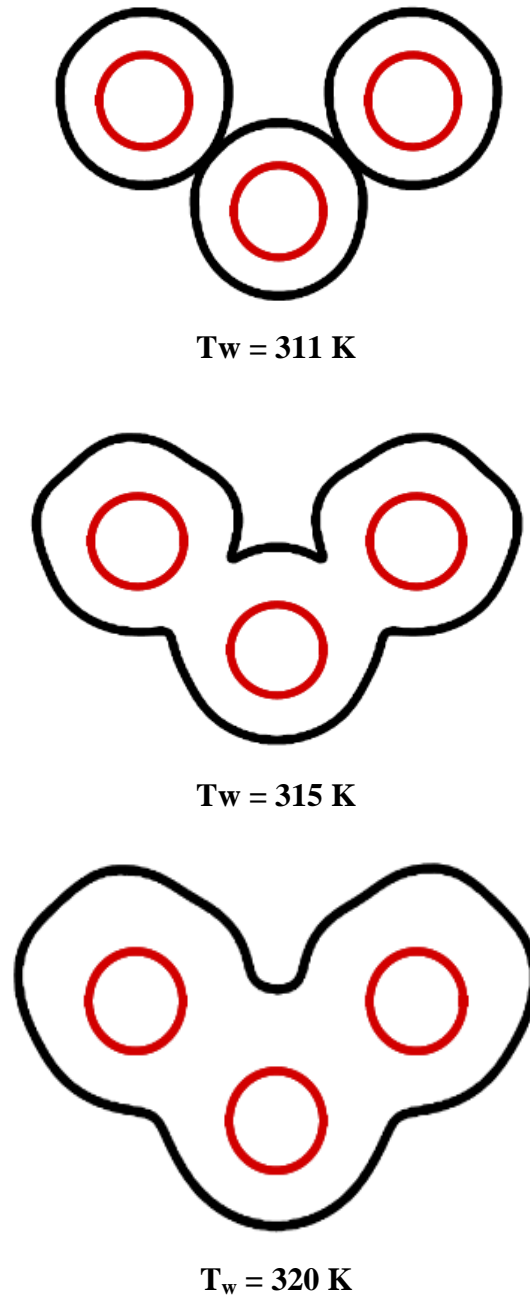


Fig. 51: Interfaces liquid-solid for various wall temperatures at $t = 100\text{s}$ (Arrangement A_1)

The plumes which rose from the top of the heated cylinders were unstable and affected the shape of the solid-liquid interface. When the circulation was sufficiently intense, the plume above cylinder 3 was influenced by the circulation between cylinders 1 and 2 and originated on the upper part of the cylinder. The plumes of cylinders 1 and 2 produced non uniform, jagged melting shapes above the upper of two cylinders.

The effect of the melting process on the streamlines, temperature contours and the solid-liquid interface for several times for arrangement (B1) is presented in Fig. 52.

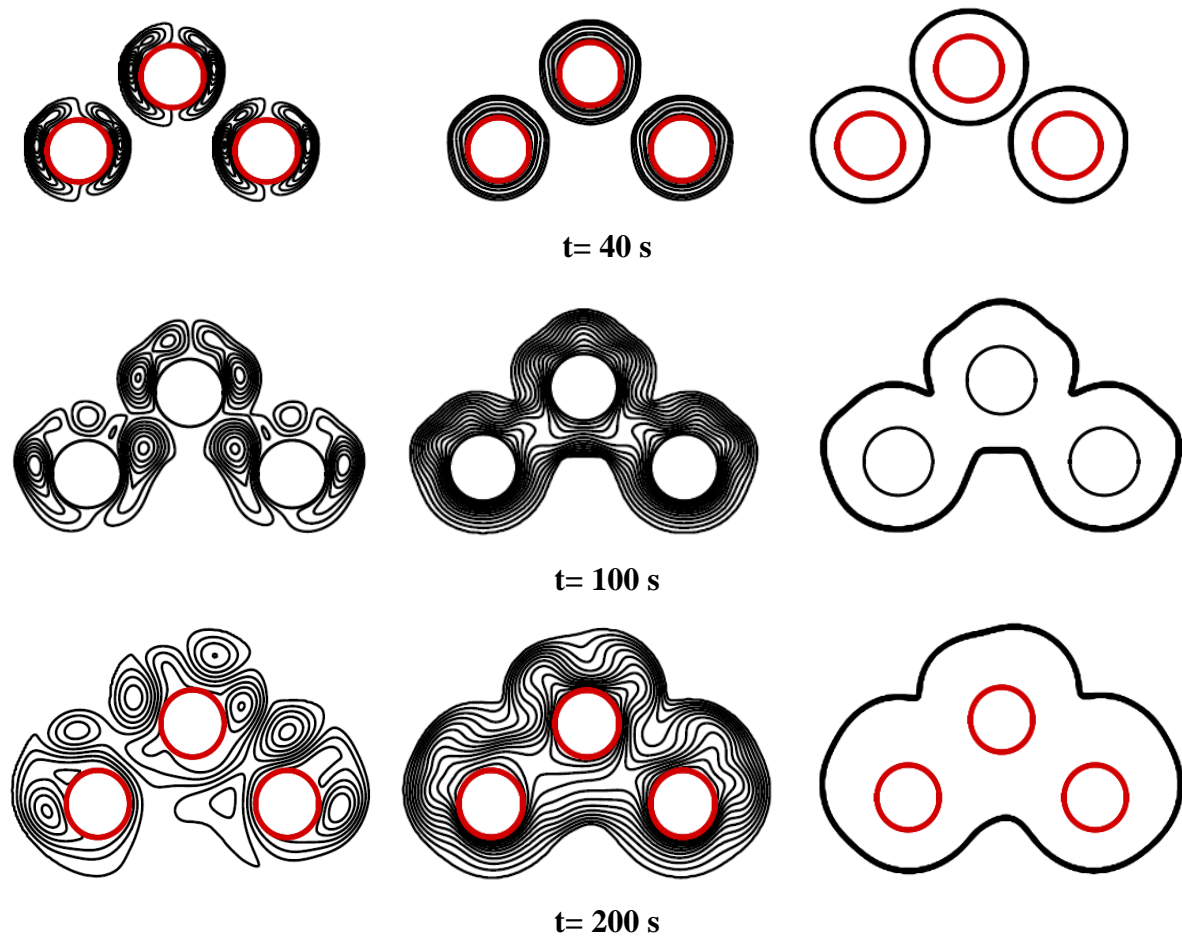


Fig. 52: Stream lines, temperatures contours and interfaces liquid-solid at various times for $T_w=315\text{K}$ (Arrangement B1)

The very important role played by natural convection in forming the melt zones is evident from these plots. Most of the melting occurs above and the sides of the heat sources with very little below. Inspection of the plots reveals that at early times the melt regions are still separated. The upward motion of the interface is driven at early times by plume which rises from the top of heated cylinder, and at later times by circulation which conveys the hot liquid to the upper part of the melt regions. It appears that the presence of natural convection reduces the melting below the cylinders compared to that which would occur if heat transfer were by pure conduction. The local Nusselt number at the hot wall is an excellent indicator of the way that convection impacts overall conduction around the three cylinders.

Figs. 53 and 54 show the evolution of the local Nusselt number at the top part of the three cylinders for several times and for the two arrangements A1 and B1.

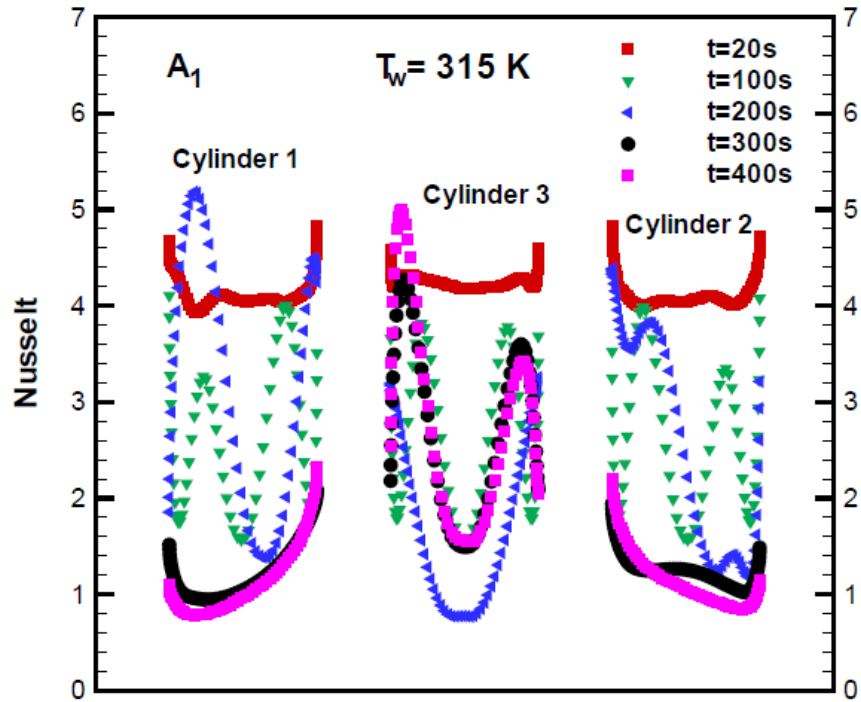


Fig.53: Nusselt number versus time (Arrangement A_1)

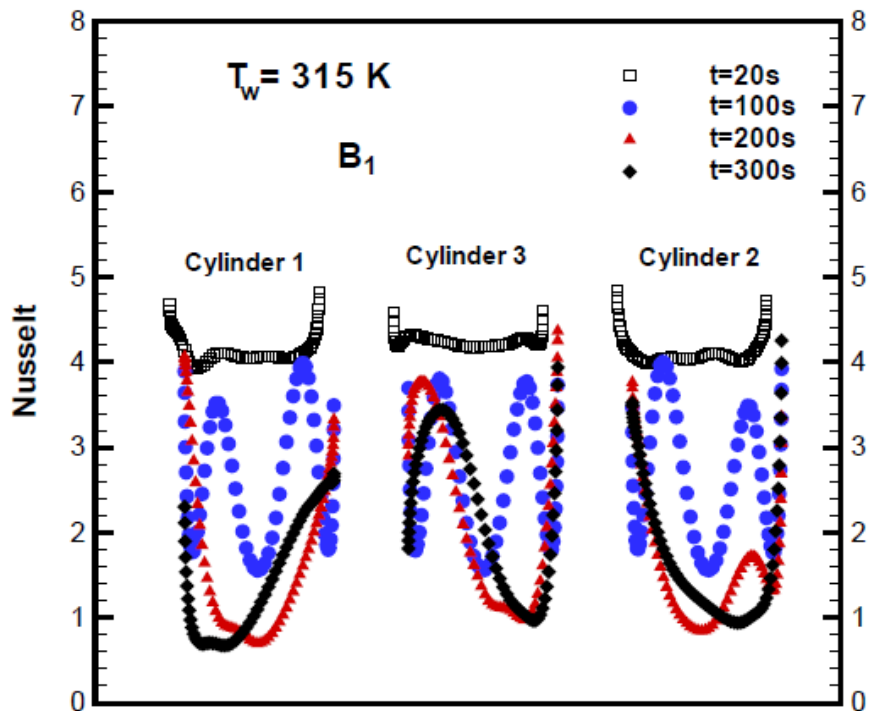


Fig.54: Nusselt number versus time (Arrangement B_1)

At first stage, when conduction predominates, the local Nusselt for both arrangements A1 and B1 is practically constant at the top part of the three cylinders. As melting continues and natural convection develops, the Nusselt number becomes non uniform. The Nusselt number leads to a constant quasi stable value at large time, even if the solid liquid interface goes on moving as melting progresses. This supposes that the processes, which happen in the neighborhood of the interface, do not bring substantial support to the overall thermal resistance to heat transfer. The variation of the local Nusselt at the top part of the cylinder can also be used to detect the oscillation of the plume in the top part of the cylinder which becomes important between 200s and 300s For arrangements A1 and B1 a continuous solid-liquid interface has already been formed while for A2 and B2 the cylinders still have separate melt zones (see Figs. 55 and 57). This is due to closer spacing of heat sources for arrangements A1 and B1 (see Table1). For arrangements A2 and B2 the melt zones extend in to the vertical and for A1 and B1 in to the horizontal directions (see Figs. 56 and 58).

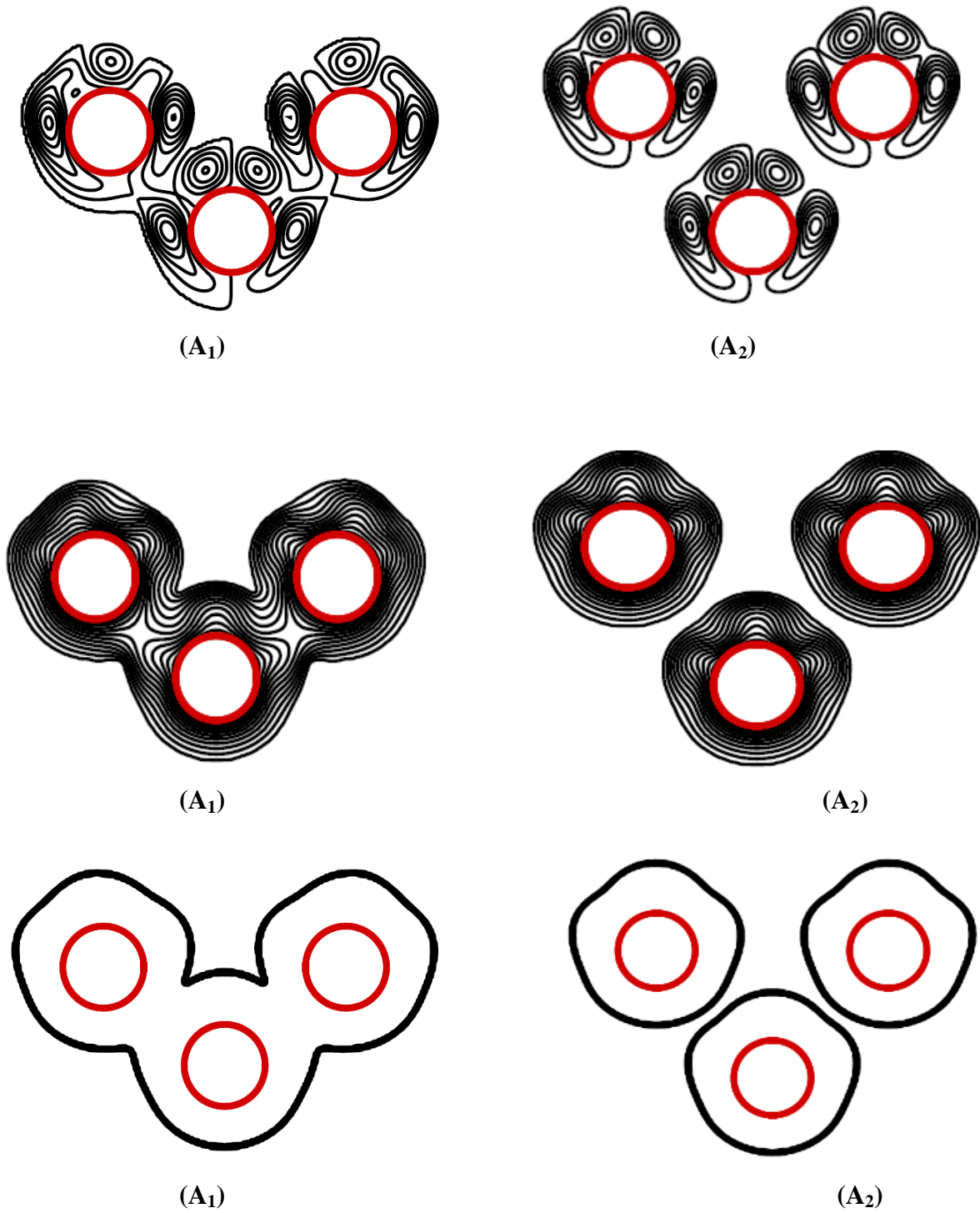


Fig. 55: Stream lines, temperatures contours and interfaces liquid-solid at $t=100s$ for arrangement A ($T_w=315$ K).

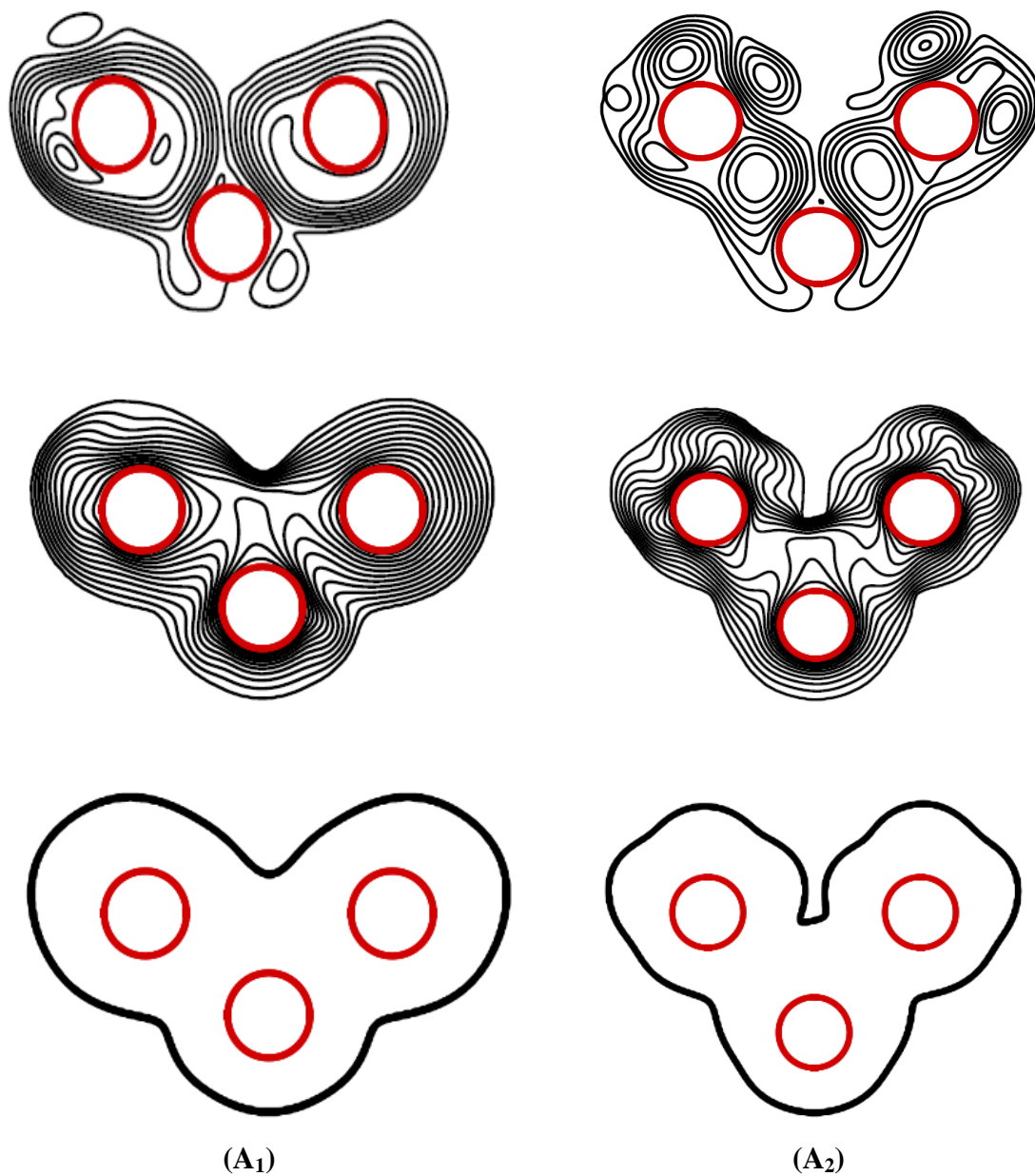


Fig. 56: Stream lines, temperatures contours and interfaces liquid-solid at $t=200s$ for arrangement A ($T_w = 315$ K).

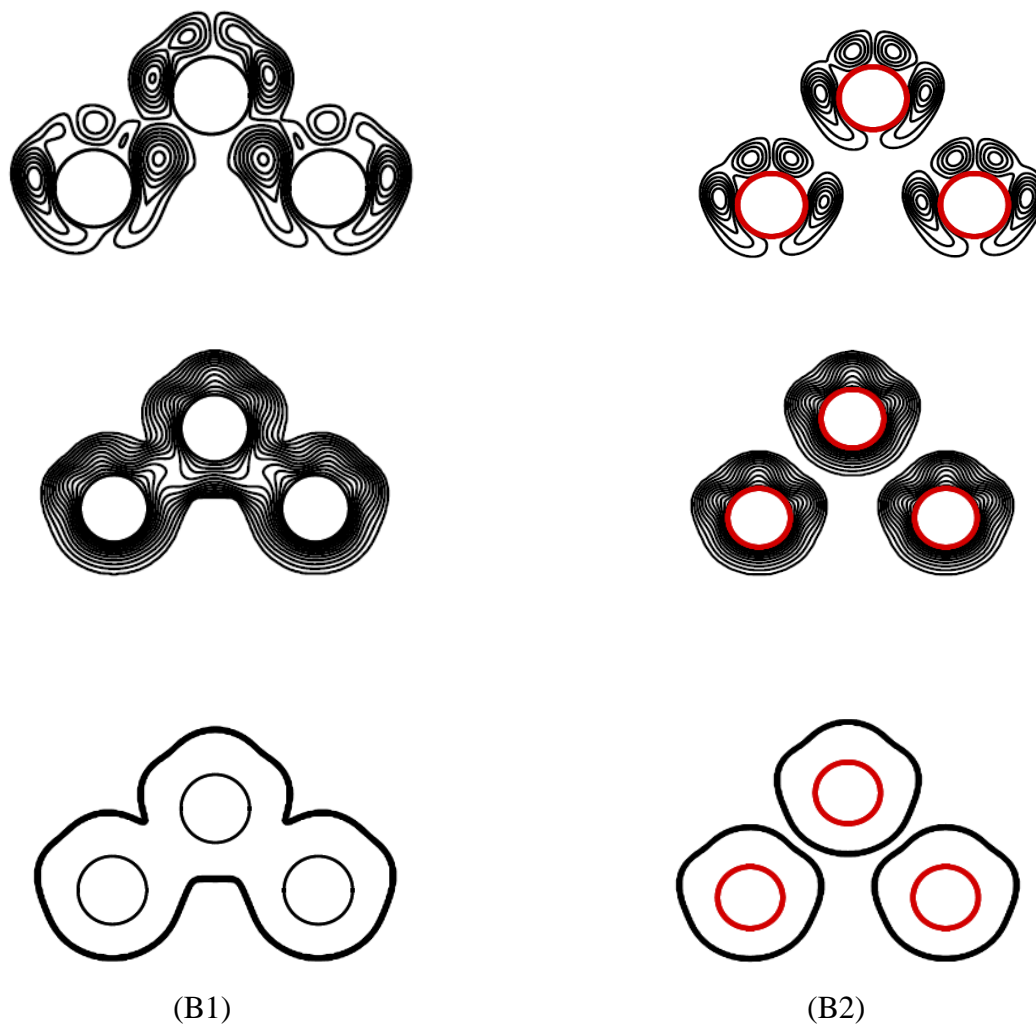


Fig. 57: Stream lines, temperatures contours and interfaces liquid-solid at $t=100s$ for arrangement B ($T_w = 315$ K).

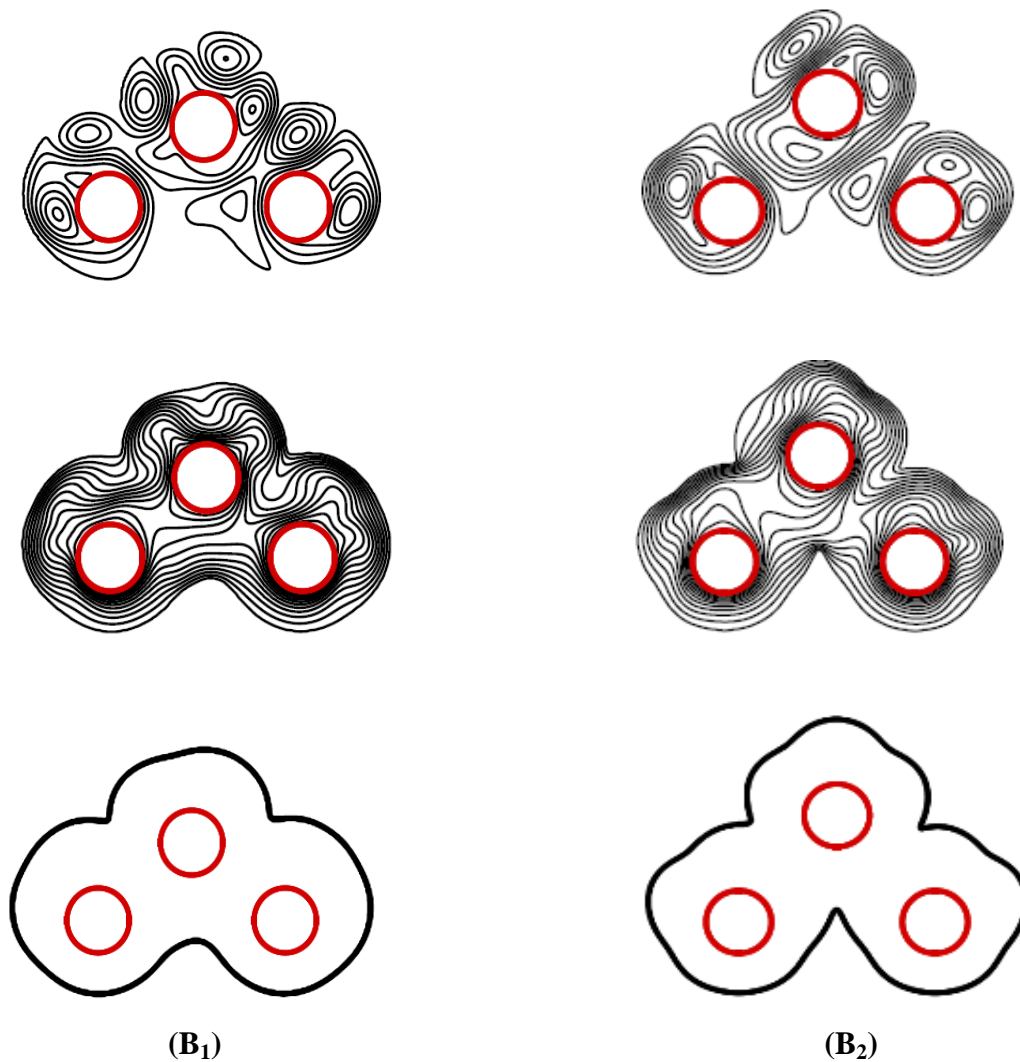


Fig. 58: Stream lines, temperatures contours and interfaces liquid-solid at $t=200s$ for arrangement B ($T_w = 315 K$).

Fig. 59 illustrates the time evolution of the total liquid fraction (ratio of volume of melt to volume cavity) versus time for the four arrangements (A1, A2, B1 and B2). From the liquid fraction versus time plot, one can get both the rate melting (slope of the tangent line at a given time) and the average melting rate (ratio of current liquid fraction and time). As can be seen, arrangements A1 and B1 proved to be more effective in melting the PCM than arrangements A2 and B2. This is due to closer spacing of heat sources for arrangements A1 and B1. We can also note that arrangement B1 is more effective than arrangement A1. This is due to the more intense natural convection circulation in the melt for arrangement B1.

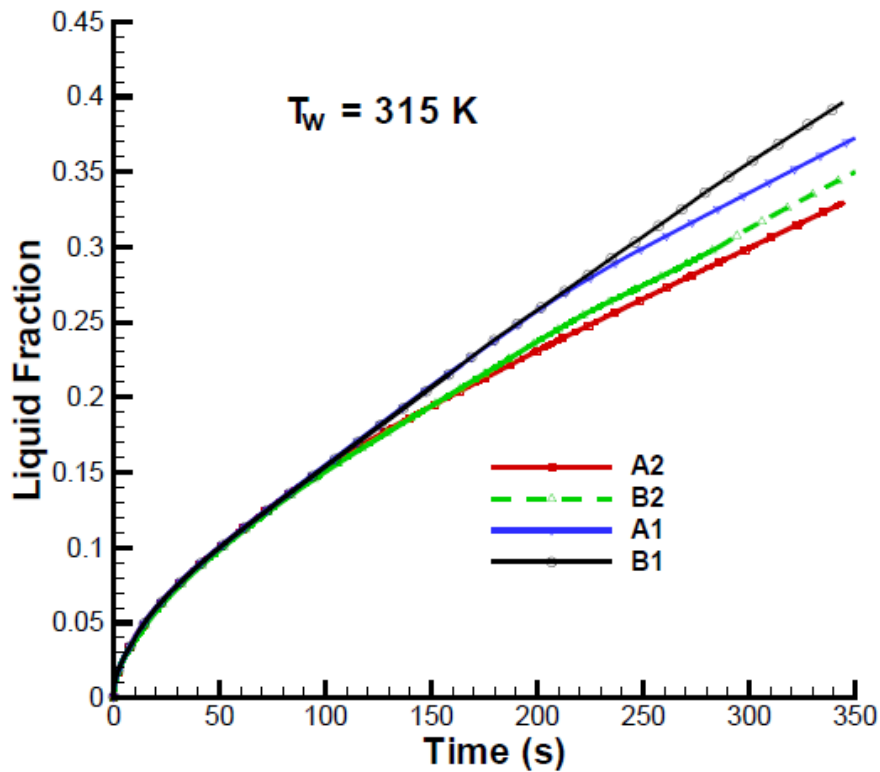


Fig.59: Liquid fraction versus time for different arrangements

3- Conclusion

Melting of PCM from multiple cylindrical heat sources has been studied numerically. It is found that after a common solid-liquid interface is formed around the cylinders, natural convection circulation around each cylinder interacts with the other cylinders to influence the melt shape. In addition to natural convection, the heat source arrangement is an important factor in determining the melt shape. For an effective utilization of a PCM in TES system the effect of natural convection and of cylinder arrangement are important and have to be accounted for in the design of such systems.

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Chapter V:

Melting Process In an Elliptical Enclosure

1- Introduction

Many numerically and experimentally studies have been carried out to study the effect of the natural convection on the phase change process in various enclosures [5-8]. The control of the melting process inside the enclosure is assumed through the conductivity of the solid PCM in the preliminary phase, and through the natural convection in the liquid region in the later phase [9-13]. These authors have also demonstrated that the thermo-physical specificities of the PCM and the geometry of the enclosure determine the aforementioned natural convection. That's why both of the form and extent of the cavity, comprising the PCM, has a considerable impact on the melting process [3]. It has been found that capsules with the elliptic geometry can create less resistance and decrease phase change time [14-15]. Given the status of the limited work on the heat transfer during melting process in elliptical capsules for different inclination angles, the present numerical study was undertaken. The problem of the melting process is formulated using the enthalpy-porosity based method. A numerical code is developed using an unstructured finite-volume method. In addition, Gallium as a phase change material (PCM) with low Prandtl number is selected.

In this study, the elliptical capsule contain solid PCM (see Fig.60) initially at temperature lower that the melting temperature T_m . The physical properties for pure gallium adopted in this research are the same as in previous studies. We have chosen the physical property values at 32°C, which is representative of the temperature range of the experiment. Since the present work is focused on the analysis of constrained melting, both the solid and liquid phases have the same density. At time $t=0$, the surface temperature of the elliptical capsule is raised impulsively to a prescribed temperature above the melting point, $T_H > T_m$.

In this study, numerical investigations were conducted using 145000 cells and the time step of 10⁻³s was found to be sufficient to give accurate results. The grid size and the time step were chosen after careful examination of the independency of the results to these parameters. The convergence is checked at each time step, with the convergence criterion of 10⁻⁶ for all variables.

2- Results and discussion

The elliptical capsule examined in this work is represented in Fig.60. The capsule has long and short semi-axes, a and b , with an inclination angle: θ .

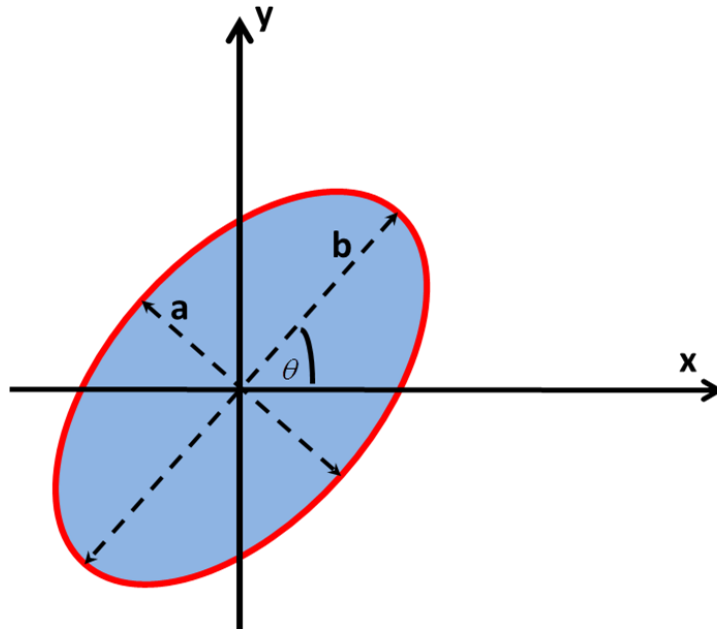


Fig 60: Schematics of the melting process in an elliptical capsule

The surface curve of the elliptical capsule is defined by the following equation:

$$\left(\frac{x}{a}\right)^2 + \left(\frac{y}{b}\right)^2 = 1 \quad (7)$$

The ellipse aspect ratio is defined as: $n = \frac{b}{a}$. Four aspect ratios are studied, $n = 1, 2, 3$ and 4. The $n = 1$ is for a circular enclosure configuration, while $n > 1$ is for elliptical enclosure configurations. To ensure conservation mass of the PCM in the all configurations, the capsules' cross section area is maintained constant. The characteristics of the flow and temperature fields are examined by exploring the effect of the aspect ratio and the inclination angle on the melting process in the elliptical enclosure.

Figs.61, 62, 63 and 64 present the melting process in the horizontal elliptical cavity ($\theta = 0^\circ$) for four aspect ratios ($n = 1, 2, 3$ and 4).

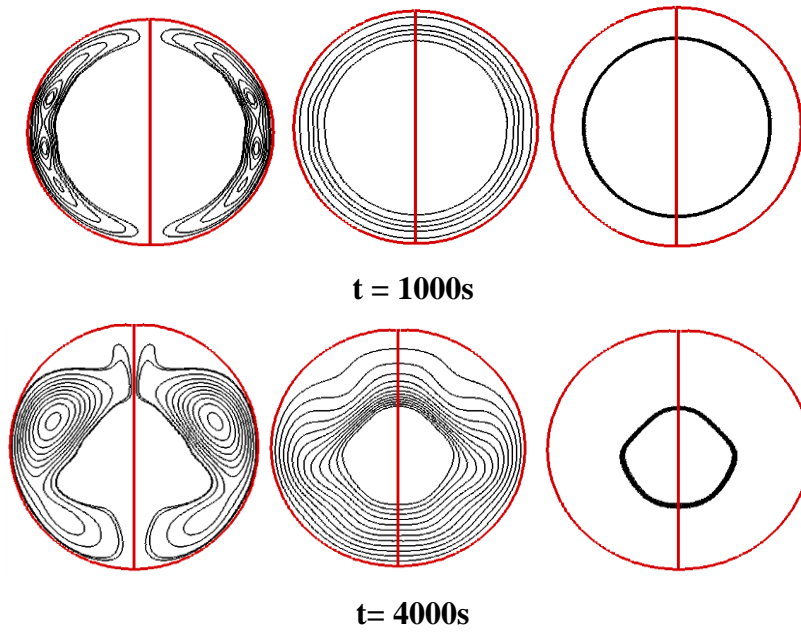


Fig 61: Stream lines, temperatures contours and interfaces liquid-solid at times 1000s and 4000s ($n = 1$; Inclination angle $\theta = 0^\circ$; $Ra = 10^4$)

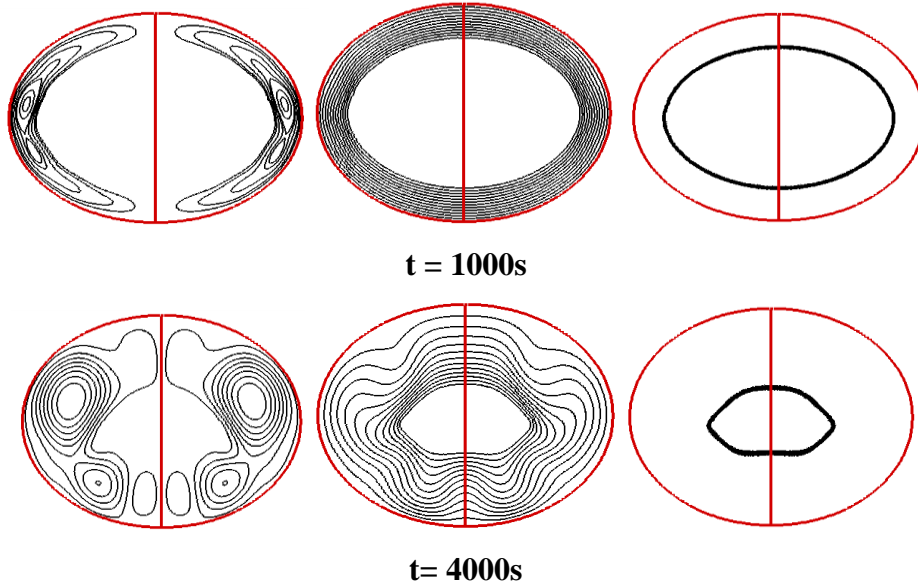


Fig 62: Stream lines, temperatures contours and interfaces liquid-solid at times 1000s and 4000s ($n = 2$; Inclination angle $\theta = 0^\circ$; $Ra = 10^4$)

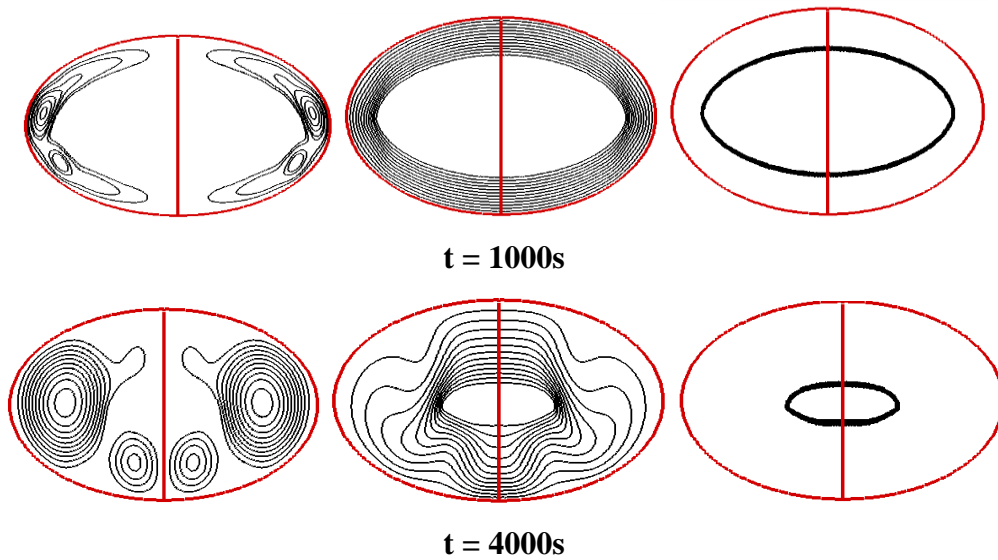


Fig 63: Stream lines, temperatures contours and interfaces liquid-solid at times 1000s and 4000s ($n = 3$; Inclination angle $\theta = 0^\circ$; $Ra = 10^4$)

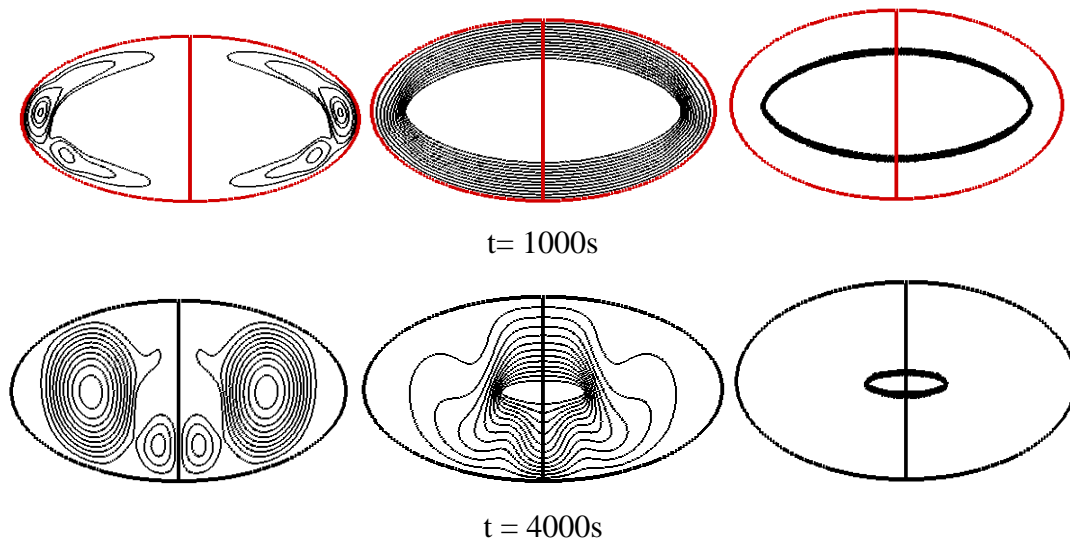


Fig 64: Stream lines, temperatures contours and interfaces liquid-solid at times 1000s and 4000s ($n = 4$; Inclination angle $\theta = 0^\circ$; $Ra = 10^4$)

The Rayleigh number is fixed at 10^4 . Firstly, the molten layer thickness is practically the same during the radius-direction of the elliptical capsule as seen in Figs.61, 62, 63 and 64. As soon as the molten layer thickness becomes very small, conduction is likely to be the main heat transfer mechanism. Past this stage, natural convection flow has an

increasing influence an important impact on the physics of the system. When the Rayleigh number is small ($Ra = 10^4$), buoyancy-driven convection almost have not great effect in the melting process. The contours levels of the temperature field feature concentric ring-like patterns and the interface at different angular locations recess almost at the same rate.

Fig.65 and 66 illustrates the temperature variation at various points inside the capsule for two aspect ratios ($n=1$ and $n=3$ respectively).

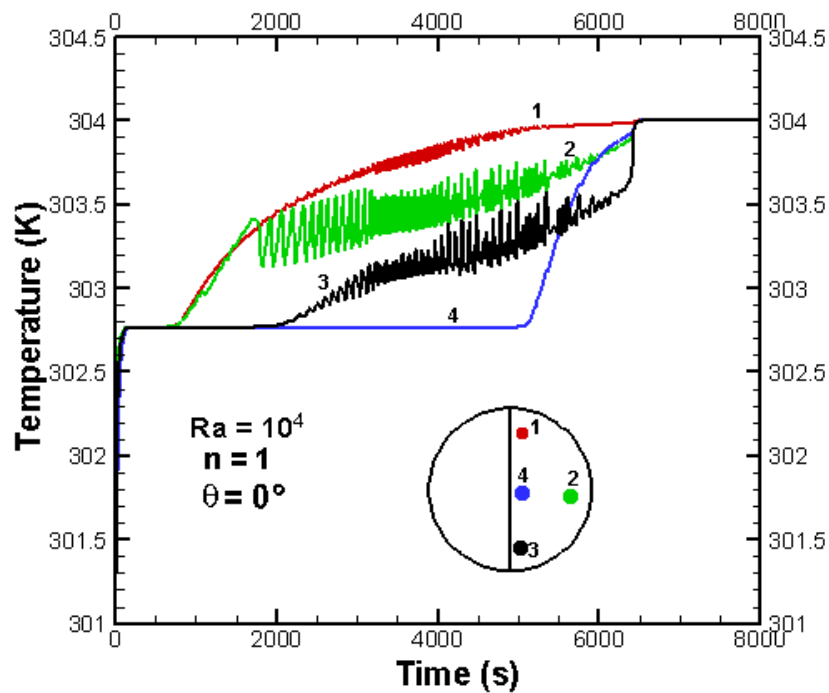


Fig 65: Temperature at various points inside the capsule ($n = 1$; Inclination angle = 0° ;
($Ra = 10^4$)

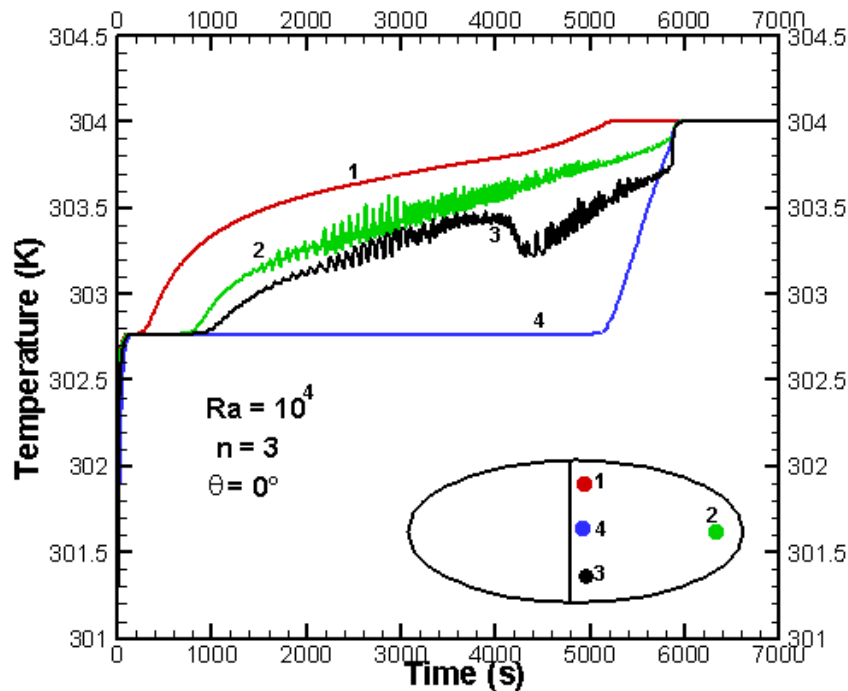


Fig 66: Temperature at various points inside the capsule ($n = 3$; Inclination angle = 0° ; ($Ra = 10^4$))

We note that inside the PCM inside the capsule includes three regions: solid region, phase change region and liquid region. For the two cases, when the temperature of each point is below is less than the melting temperature, heat is transferred by conduction through the solid PCM. When the temperature of capsule wall reaches the melting temperature, the melting process inside the PCM will begin. Once the solid-liquid interface passes each point, we can note that there will be a sharp increase in liquid temperature at these points. Due the stable nature of the liquid layer that contains the point 1, no strong convective motion that would cause chaotic temperature readings are observed. During the melting process, some temperature fluctuations are noticed in temperature histories at points 2 and 3. These fluctuations indicate the presence of chaotic and vortical flow structures in the liquid region at points 2 and 3. The point 4 which is placed at the center of the capsule is the last point at which the temperature readings °.

The flow behavior during the melting process for four aspect ratios ($n = 1, 2, 3$ and 4) is given in Fig.67, 68, 69 and 70 for $Ra = 10^6$.

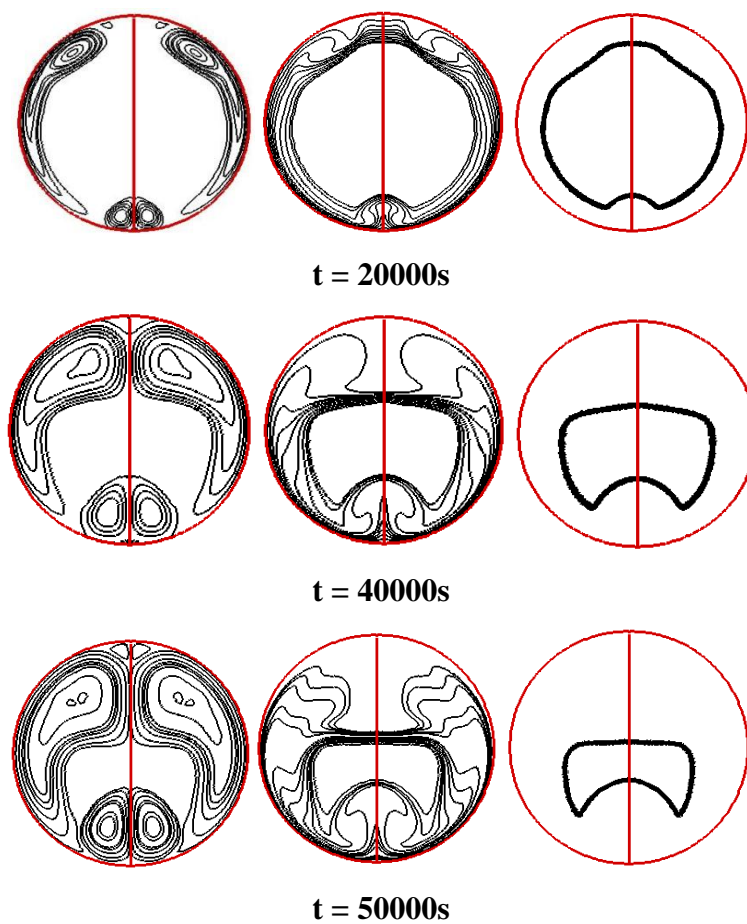


Fig 67: Stream lines, temperatures contours and interfaces liquid-solid at times 20000s, 40000s and 50000s ($n = 1$; Inclination angle = 0° ; $Ra = 10^6$)

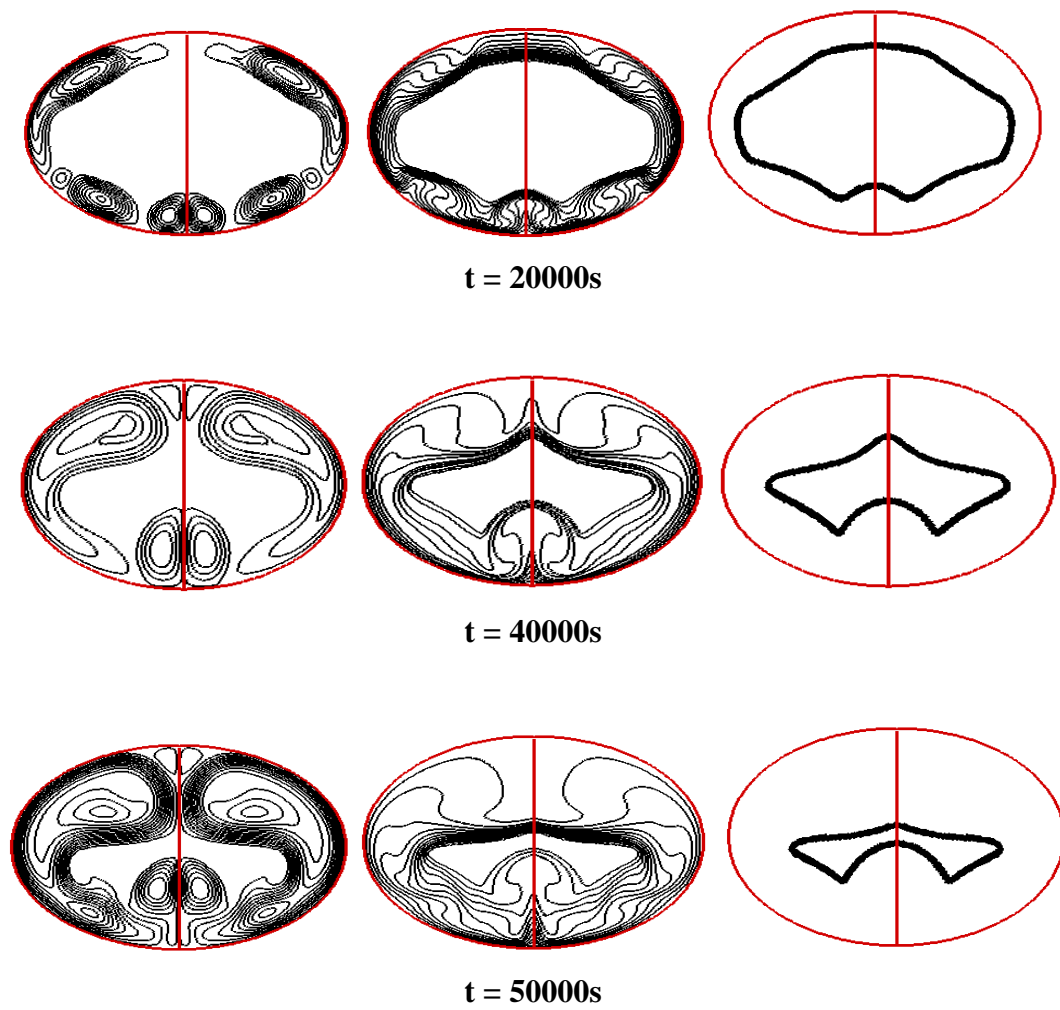


Fig 68: Stream lines, temperatures contours and interfaces liquid-solid at times 20000s, 40000s and 50000s ($n = 2$; Inclination angle = 0° ; $Ra = 10^6$)

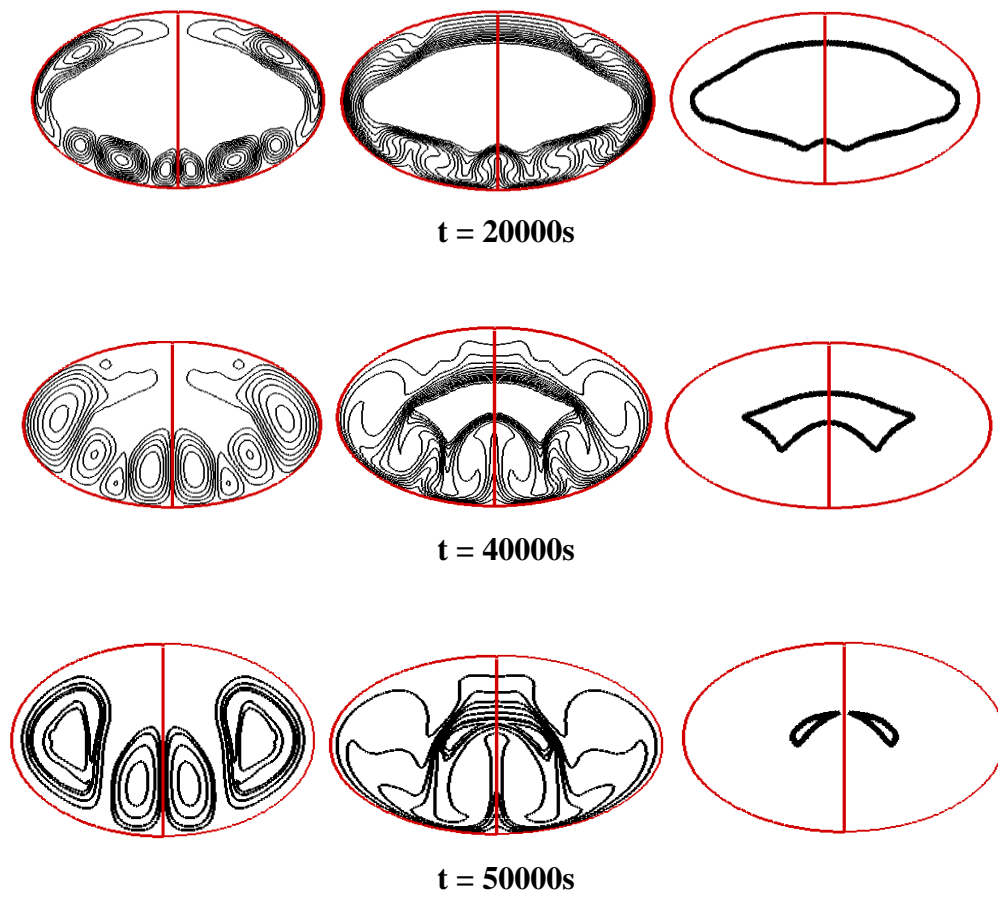


Fig 69: Stream lines, temperatures contours and interfaces liquid-solid at times 20000s, 40000s and 50000s ($n = 3$; Inclination angle $\theta = 0^\circ$; $Ra = 10^6$)

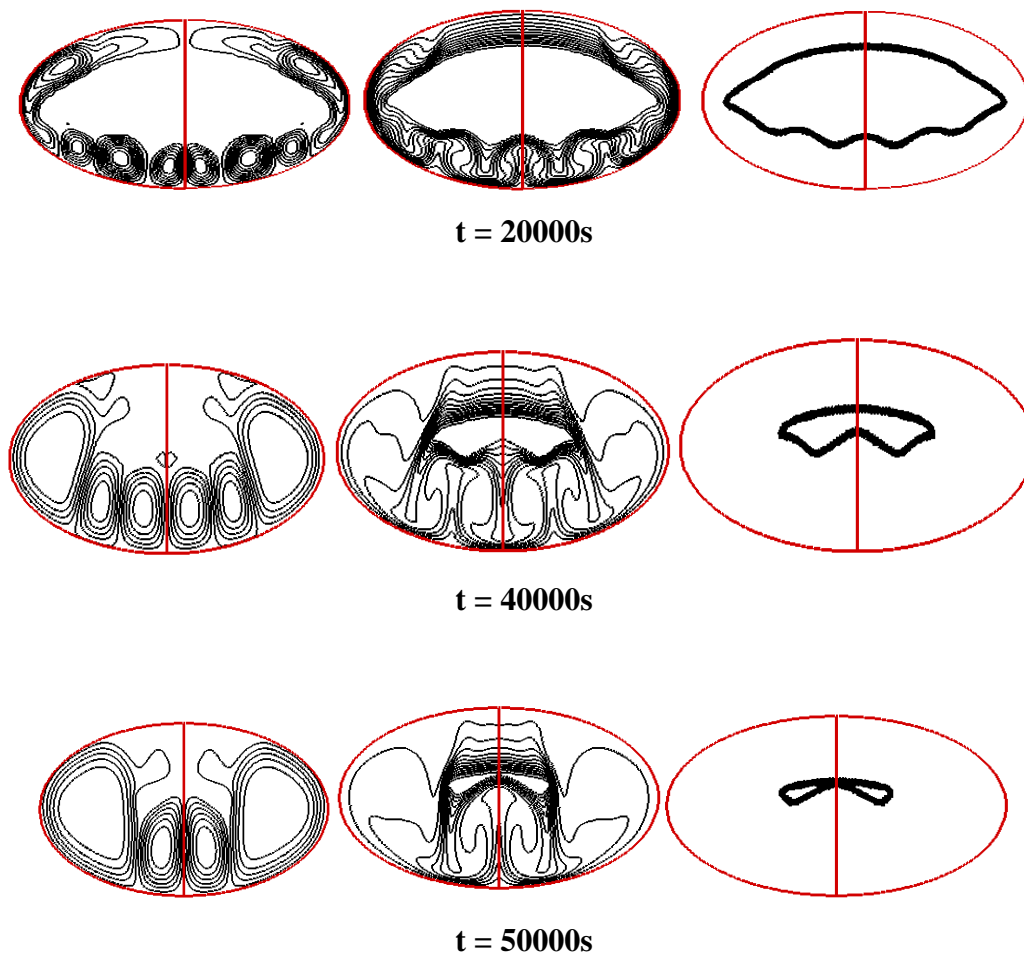


Fig 70: Stream lines, temperatures contours and interfaces liquid-solid at times 20000s, 40000s and 50000s ($n = 4$; Inclination angle $\theta = 0^\circ$; $Ra = 10^6$)

After the conduction dominating phase, follows a complex structure of the fluid dynamic which is distinguished by a multi-cellular flow patterns. The size of the liquid region determines the number and the size of the cells. Natural convection intervene as a consequence of the warm liquid PCM rises along the hot wall while the cooler liquid in the center flows down to replace the warmer fluid. This generates an unstable fluid circulation inside the capsule. Therefore, the top region of the PCM melts faster than the bottom region. As the liquid layer increases, the number of rolls decreases. When the liquid fills nearly 50% of the elliptical capsule, two counter-rotating rolls are obtained in the bottom region. This phenomenon recalls the Bénard convection due to the thermal instability because the solid-liquid interface and the elliptic bottom wall can be considered as two flat plates as far as the liquid gap between these two surfaces is narrow enough. This phenomenon leads then in the

sequel of calculations to the development of the abovementioned roll cells. The establishment of these vortices influences the melting kinetics of the whole system. Especially the heat transfer in the lower part of the melting gap is greatly improved which results in a “moonshaped” melting contour. We can also observe that the flow in the liquid zone remains in the stable state. The Bénard convection presents an orderly behavior without been affected by the base flow. So the waviness of the bottom part of the PCM can be explained by the temperature fluctuation at this region and the expedited melting caused by the penetration of multi-cellular roll structures.

Fig.71 and 72 show the temperature variation at different points within the elliptical enclosure for two aspect ratios ($n=1$ and 3 respectively).

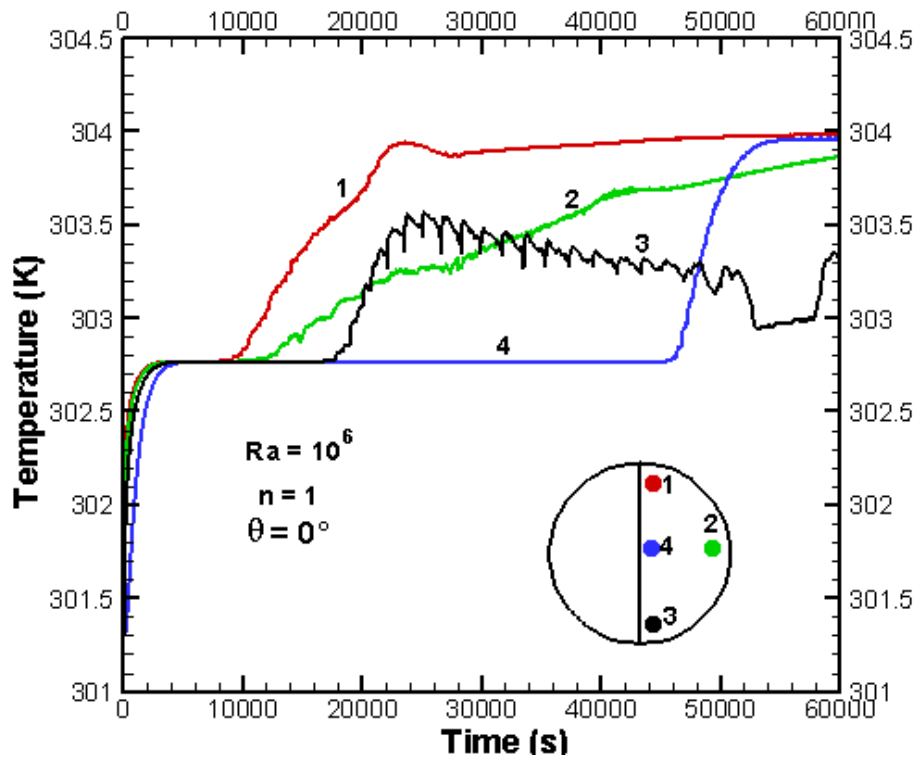


Fig 71: Temperature at various points inside the capsule
($n=1$; Inclination angle = 0° ; $Ra = 10^6$)

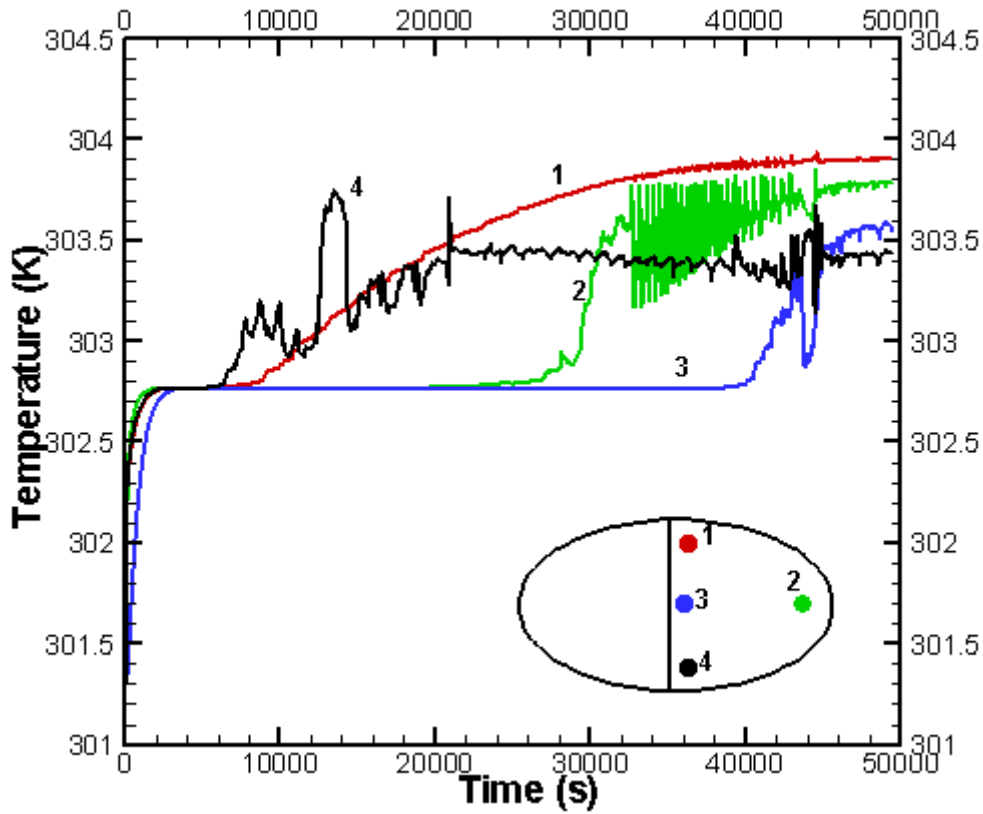


Fig 72: Temperature at various points inside the capsule
($n=3$; Inclination angle = 0° ; ($Ra = 10^6$))

We note that during the phase change process, some temperature variations are detected in temperature histories at points 2 and 3. These fluctuations variations confirm the constitution of an unstable fluid layer that supports disordered variations and is the cause of waviness of the bottom of the PCM. Due to the stable nature of the liquid layer that contains the point 1, no strong convective motion that would cause chaotic temperature readings are observed.

Fig.73 and 74 show the time evolution of the total fraction of the liquid in the elliptical cavity for different aspect ratio ($n = 1, 2, 3$ and 4) for $Ra = 10^4$ and $Ra = 10^6$.

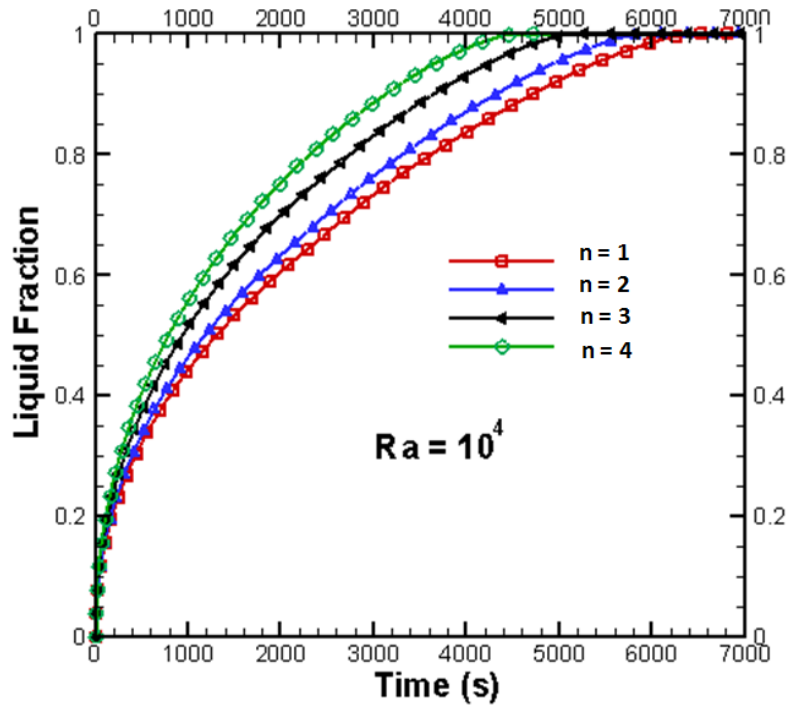


Fig 73: Comparison of the liquid fraction variations versus time for different aspect ratio (Inclination angle = 0° ; $Ra = 10^4$)

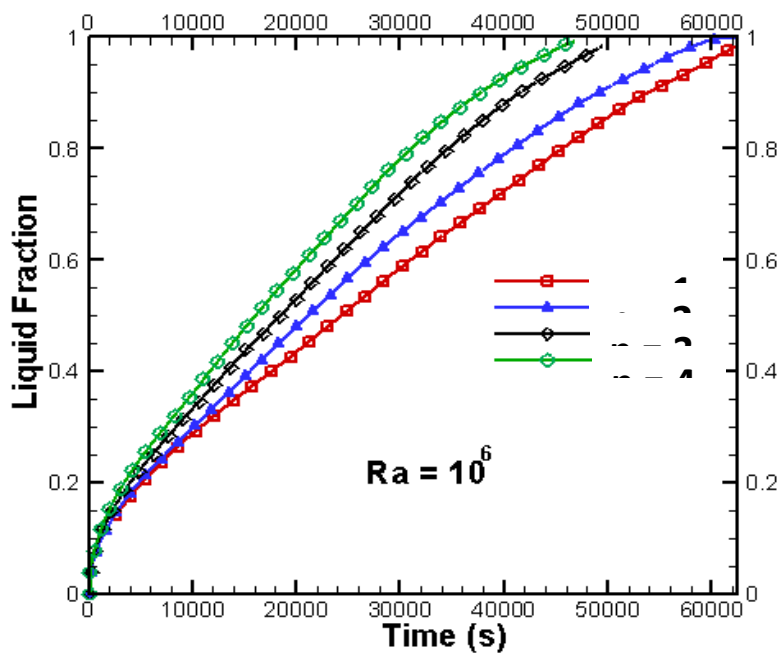


Fig 74: Comparison of the liquid fraction variations versus time for different aspect ratio (Inclination angle = 0° ; $Ra = 10^6$)

From the liquid fraction versus time plot, one can get both the rate melting (slope of the tangent line at a given time) and the average melting rate (ratio of current liquid fraction and time). Figs. 73 and 74 indicate that increasing the aspect ratio has a positive effect on the melting rate of the PCM. We can also note that the melting time decreases by increasing the aspect ratio of the elliptical enclosure. Therefore, the ellipse's ratio can be utilized to regulate the melting process in elliptical capsules.

3- Conclusion

Melting process in an elliptical heat exchanger with different aspect ratios ($n = 1, 2, 3$ and 4) filed by gallium as PCM has been considered numerically. The characteristics of the flow and temperature fields are examined by exploring the effect of the aspect ratio; it has been found that increasing the aspect ratio has a positive effect on the melting rate of the PCM and reduces the melting time of the elliptical enclosure.

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Conclusion And Perspectives:

Heat transfer process in phase change materials was the objective of the present thesis, different encapsulation geometries are investigated, and the enhancement techniques employed in PCMs to effectively charge and discharge latent heat energy was the main objective of the present work. Modelisation and simulation of phase change material are investigated to assess the effects of different parameters such as the mass flow rate of the heat transfer fluid (HTF) and the inlet temperature.

The first chapter concerned the physical modeling and mathematical formulation of the enthalpy porosity approach for heat transfer problems associated with melting and solidification of phase change material. One-dimensional Stefan Problem for Pure Materials was then presented.

Chapter 2 was devoted to study the melting process in a rectangular cavity. The effect of titling the rectangular cavity on the melting process of phase change material is studied; results for different inclination angels are presented and discussed. Melting of PCM over wavy surface is then suited; the effects of the amplitude and the number of wavy surface on the rate of melting of a scare wavy cavity. Both horizontal and vertical wavy surfaces are studied and their results are compared to plat surface. The main conclusion was that wavy surfaces are more efficient in heat transfer because of their capability to promote fluid motion near the surface.

In chapter 3 cylindrical enclosure was studied, in the first part both vertical and horizontal cylindrical capsule filed with PCM was treated; the enthalpy-porosity method was employed. The effect of introducing fins embedded in PCM in horizontal cylinder was then studied. The man conclusion was that the use of extended surfaces increases the rate of melting in comparison to that without fins, and Orientation of fins with respect to the gravitational field on a horizontal cylindrical is interesting for taking full advantage of natural convection effects, to enhance melting and not to create quiescent melt regions where natural convection circulation may be suppressed.

Chapter 4 investigated the case of melting of PCM around horizontal cylinders. A relevant consideration in such systems is the effective utilization of the PCM by an optimum arrangement of tubes through which the working fluid is circulated. This chapter concludes that, the heat source arrangement is an important factor in determining the melt shape. For an effective utilization of a PCM in TES system the effect of natural convection and of cylinder arrangement are important and have to be accounted for in the design of such systems.

In chapter 5 the melting process in an elliptical enclosure filed with PCM was studied, the results show that increasing the aspect ratio has a positive effect on the melting rate of the PCM. The aspect ratio of the elliptical enclosure seems to have a considerable effect on the

melting time. Therefore, the ellipse's ratio can be utilized to regulate the melting process in elliptical capsules.

The present work can be continued by studying other geometries encapsulations; actually we are investigating the effect of titling the elliptical enclosure on the melting process and studying how the inclination angel affects the melting time of the PCM into the elliptical inclined encapsulation.